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Interview with Jean Petitot

Jacqueline Léon, Sylvain Loiseau

Abstract. From the 1960s onwards, the mathematician René Thom (1923-2002) carried out important contributions to the mathematical modelling of morphogenesis (analysis of forms). The proposed concepts (singularity, structural stability, catastrophe, bifurcation) were re-used in several social sciences, particularly in linguistics. They allowed a Gestalt-like approach, in opposition to the then dominant logico-combinatorial ones, and met some cognitivist trends and connectionist models. Jean Petitot was the first to show interest in the application of Thom's work to linguistics and he developed many studies accordingly.

This article is based on an interview between J. Petitot, J. Léon and S. Loiseau held on the 27 of September, 2014. While preserving the oral style of free conversation, it also includes references, developments and mathematical statements added by the authors.

I. Jean Petitot's biography

After preparatory classes at Louis-le-Grand high school, I entered the École Polytechnique in 1965 where I graduated in 1968. Impassioned by research, I joined the new Centre of mathematics my professor Laurent Schwartz had just created, and I learned algebraic geometry (with Grothendieck's disciple Jean Giraud) and differential geometry. As I investigated singularity theory[1], I met René Thom, one of the leading specialists of the field, who had restructured it entirely since the middle of the 1950s[2].

Besides, I was much interested in structuralism, in particular Claude Lévi-Strauss, whose lectures at the Collège de France I already attended when I was very young, around 18-19 years old. It is through Lévi-Strauss that I discovered Roman Jakobson. At that time, I did not see any link between structuralism and mathematics.

At the end of the 1960s, René Thom started to circulate the manuscript of *Stabilité structurelle et morphogenèse* (published in 1972). This book, which was focused in biology, also explained why the scope of the morphogenetic approach[3] went far beyond biology and could apply to structuralism in general. Thom had much discussed with Conrad Hal Waddington (1905-1975), an eminent specialist of embryogenesis, and he had benefited from Jakobson's strong support. His book, proposing a mathematical structural approach of biology, became very controversial. Its media audience owed much to Christopher Zeeman (University of Warwick), who coined the term "catastrophe theory" and turned it into a very general methodology whereas Thom's objectives were more focused[4].

In 1969 I discussed with René Thom his applications of singularity theory to structuralism. According to what he told me, I was the first young mathematician of my generation to do it.

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[1] cf. *infra*.

[2] René Thom (1923-2002) was a mathematician and a former student of the Ecole Normale Supérieure. He had lectured in Grenoble, Strasbourg and, near Paris, at the Institut des Hautes Etudes Scientifiques, until 1990. He received the Fields medal in 1958 for his work on differential topology. He developed mathematical models of morphogenesis popularized under the name of "Catastrophe theory" (cf. *Stabilité structurelle et morphogenèse*, 1972, InterÉditions, Paris).

[3] Morphogenesis studies the formation processes of complex forms, in particular those of life.

[4] Christopher Zeeman (born in 1925) founded the Department of Mathematics and the Research Centre in Mathematics of the University of Warwick in 1964. In 1969-1970, during a sabbatical year in Paris, he discovered René Thom's Catastroph theory. Next, he largely contributed to the notoriety of this theory by providing it with many applications in various fields, in particular in social and behavioural sciences (cf. Isnard C. A. & Zeeman E. C. (1976). Some models from catastrophe theory in the social sciences". In: Collins L. (ed.) *Use of Models in the Social Sciences*, Tavistock, London, pp. 44-100).

These discussions filled me with enthusiasm. After having hesitated between pure mathematics (I then had a position at CNRS, Centre National de la Recherche Scientifique) and modelling, I accepted in 1971 a position at the Center of Analysis and Social Mathematics (CAMS) of the 6th section of the EPHE (École Pratique des Hautes Études), which would become later the EHESS - Ecole des Hautes Etudes en Sciences Sociales). I was recruited at the EPHE thanks to the support of Lévi-Strauss, Fernand Braudel, the director of the 6th section, who believed in the role of mathematics in the social sciences, and Charles Morazé, my former professor at the Ecole Polytechnique.

Having joined the EPHE, I naturally got in touch with some structuralists, including A.J. Greimas. Greimas made an announcement in his seminar and some young colleagues got interested. I thus met Jean-François Bordron, Frédéric Nef, Paolo Fabbri, Jean-Claude Coquet, Per Aage Brandt, François Rastier, Claude Chabrol, and later Jacques Fontanille, Ivan Darrault, Jean-Jacques Vincensini and several other semioticians. Greimas did not have a very strong institutional position and his disciples had rather difficult careers, but he compensated for this fragility by his exceptional dimension.

Thanks to Paolo Fabbri who put me in touch with Umberto Eco, I spent one year in Bologna where I wrote a part of my Habiltation thesis ("thèse d'état"). I spread Thom's work on structuralism in the international semiotics community, in Bologna of course, then within Per Aage Brandt's group in Aarhus in Denmark and in Toronto. I very early discussed with Jean-Pierre Desclés who worked at the time with Antoine Culioli. Thus, I became involved in a social sciences community where I could use my double competence in mathematics and semiotics.

Later on, I remained primarily at the CAMS. I defended my "thèse d'état" in 1982[5], and, until 1985, I remained focused on applications of Thom's morphodynamical models (*i*) to phonetics (*Les catastrophes de la parole. De Roman Jakobson à René Thom.* Maloine, Paris, 1985), (*ii*) to elementary structures in semiotics, (*iii*) and to theories of actantial syntax, in particular case grammars (*Morphogenèse du sens. Pour un schématisme de la structure.* PUF, Paris, 1985).

Afterwards, I became more and more interested in cognitive neurosciences. In 1986, I joined a team of cognitive sciences which had just been created by Daniel Andler in a lab of the Ecole Polytechnique, the CREA (Centre de Recherche en Epistémologie Appliquée) founded in 1982 and directed by Jean-Pierre Dupuy. A little later, Michel Imbert, a specialist in neurosciences, created the first DEA (Diplôme d'Etudes Avancées - a post-graduate diploma) of cognitive sciences, and I became actively involved there in this new context, which led me establishing footbridges with American, in particular Californian, cognitivism.

II. René Thom's contributions

Q.: You said you were initially interested in Thom's work in mathematics, in particular in his work on the theory of singularities you were working on. Then, you were interested in his work on structuralism in linguistics. Can you tell us about

[5] *Pour un Schématisme de la Structure: de quelques implications sémiotiques de la théorie des catastrophes*, thèse d'état defended in 1982 at the École des Hautes Études en Sciences Sociales, Paris.

Thom's contributions in these fields? To start with, what is his theory of singularities?

Singularity theory aims to study, analyze and classify geometrical structures of a specific type, which are called "singular" because they are not "regular". One considers a class of objects for which (*i*) the opposition between local and global properties is meaningful, and (*ii*) there are standard "simple" objects whose structure is "trivial". Then, one calls "regular" the objects that are everywhere locally simple (or "locally trivial"), even if they can be globally very complex and not trivial at all. There exist singularities when, locally, the considered object is not regular.

For example, let us consider surfaces and define a regular point as a point where the surface admits a tangent plane. Let us take a cone: apart from the vertex there is a tangent plane at every point and the cone is thus locally regular. But the vertex does not have a tangent plane and is thus a singular point. And as it is the only singular point in a small neighbourhood, it is said to be an isolated singularity.

One is immediately confronted with a theoretical problem: how to classify the singularities? One can observe for example that there exist points that are more or less singular. Consider for example a roof: apart from the ridge, points are regular. The points of the ridge are singular but are not isolated singularities since the whole ridge consists of singular points. The cone apex is more singular, it has a larger "degree" of singularity than the ridge of a roof.

One can thus establish a hierarchy of singularities and it is necessary to build a battery of theoretical concepts to analyze all the possibilities.

The interest of singularities — it was one of Thom's great ideas — is that if all the local singularities of an object are known, then the object can be globally known qualitatively. The singularities concentrate the qualitative information on the objects. I give an example, introduced by Moebius in the 19th century, but known for centuries by sculptors. A good way of understanding a three-dimensional form is to cut it out in two-dimensional slices and to consider these successive slices. Take a torus (considered vertically) and cut it out in horizontal slices of increasing height (see fig. 1). If the cutting plane is too low, you do not meet the torus. At a certain time you meet the torus at a first singular point (a minimum). When you still go up, you get circles. You still go up and you meet another singular point (a saddle): the section has the form of an 8. The following level lines contain two circles, then again one 8, then only circle, and finally you reach the point at the apex of the torus (a maximum).

Conversely, if by cutting out a surface in slices you meet four singular points of this type (a minimum, two saddles, a maximum), then the surface is topologically a torus. Topologically these four singular points (with their type) characterize a closed surface with a central hole. The fact that the list of the singular points with their type define the object topologically is called Morse theorem.

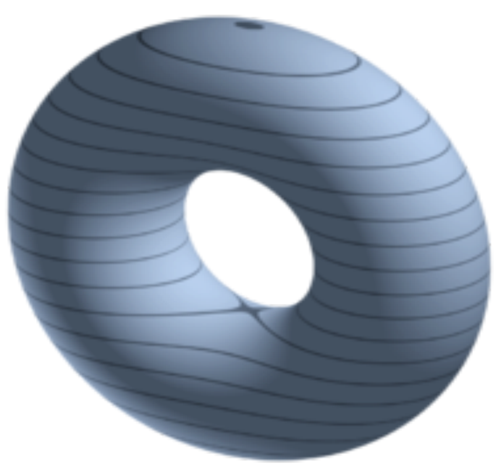

Figure 1. A torus with its level lines. Morse theorem

(source: http://fr.wikipedia.org/wiki/Théorie_de_Morse)

The classes of structures you can analyze with such methods are of a great diversity. You can look for example at what are called differentiable manifolds which generalize the concept of surface; you can also look at maps between spaces. In all these cases, you consider classes of objects and you want to study their possible non-trivial local properties.

The levels of structure can be very different from one another. You can consider topological objects (a very low level of structure but where the concept of continuity has a meaning nevertheless); or objects having more rigid properties. For example, if you take an orange peel and you try to crush it on a plane, as it is not elastic, it tears. This is a metric property: at the metric level, the sphere has curvature whereas the plane does not have any. Thom focused on the level of structure known as “differentiable”, which is intermediate between topological and metric levels and means that you can take as many derivatives as you want of the functions describing the objects.

Q: What are Thom’s contributions in linguistics?

To understand Thom’ contribution in the fields of semiotics and linguistics, it is necessary to come back to the specific notion of structure in linguistics, which concerns the mereological problem: how totalities can be organized with constituents, relations and transformation rules between constituents, and show an organization which is more than the sum of their constituents.

There are many fields where mereological structures can be encountered: grammatical rules and syntax (whatever the theories), but also, in psychology, visual perception spatial objects linked by spatial relations; in biology, the constituent structure of organs; in chemistry, the molecules where atoms are linked by their valence electrons, etc. These structures have been pinpointed for a long time, but, until recently, the adequate mathematics to model them in biology and in linguistics was completely missing (in chemistry it is only with quantum mechanics that they could be modelled).

This is why the issue concerning structures refers to formalization and modelling. In linguistics, the mathematical models used are multiple but generally rest on formal tools, that is algebraic, combinatorial and logical tools (Chomsky, Shaumyan, Montague, etc). In other fields, like visual perception and biological morphogenesis, structures are interpreted in a much more geometrical, topological and dynamical manner, as organized forms and Gestalts. The concept of structure is no longer algebraic and logico-combinatorial but morphological and dynamical, "morphodynamical" as I like to say.

The problem of a topological and morphodynamical mathematical theory for forms, primarily in biology and perception, is fundamental and extremely old. If we look back in history, there was a rather good theory in Aristote (cf. homeomeres and anhomeomeres in *The Parts of the Animals*) but it was completely eliminated by modern Galileo-Newtonian physics.

As André Robinet brilliantly showed, Leibniz was obsessed all his life by the antinomy thus created: one needs neo-Aristotelian concepts to work out a theory of form, but those seem to be incompatible with mechanist physics[6]. To overcome that antinomy, Kant had to write his third Critique, *The Critique of Judgment.* After him, many philosophers and scientists raised these issues. But these remained open until the 1960-70s when one suddenly saw flowering several

[6] See André Robinet (1986) *Architectonique disjonctive, Automates systémiques et Idéalité transcendantale dans l'oeuvre de G. W. Leibniz*, Paris, Vrin. See also J. Petitot (1999). "Le troisième labyrinthe: dynamique des formes et architectonique disjonctive", *L'actualité de Leibniz: les deux labyrinthes* (D. Berlioz, F. Nef eds), *Studia Leibnitiana Supplementa*, 34, 617-632, Stuttgart, Franz Steiner.

radically new theoretical proposals: Thom and Zeeman with catastrophe theory, Ilya Prigogine with dissipative structures[7], Hermann Haken with synergetics[8], Henri Atlan with self-organization[9].

It is Thom who introduced the deepest mathematical tools. The only precedent had been, about fifteen years before, that of Turing who, just before his death, had been interested in morphogenesis and had introduced the first models explaining the emergence of forms and patterns in biochemical substrates using reaction-diffusion equations[10].

In the late 1960s, one thus started to have an idea of how the old problem of a theory of forms could be apprehended. Thom then introduced, in a very radical (and very controversial) way, the assumption that these morphodynamical tools could be transferred from biological morphogenesis to structural linguistics and semiotics. As a result, he found himself at the very heart of a linguistic debate which had a rich history: that of the opposition between gestaltic views (Guillaume, Tesnière, etc.) and formal views (Chomsky, etc.). Some linguists, like Hansjakob Seiler and Bernard Pottier, were enthusiastic. Others, like the Chomskyans, were more careful, even hostile.

[7] See for example Ilya Prigogine, Isabelle Stengers (1979) *La Nouvelle Alliance. Métamorphose de la science*, Paris, Gallimard.

[8] H. Haken (1981) *The Science of Structure: Synergetics* (Van Nostrand Reinhold).

[9] H. Atlan (1972/1992) *L'Organisation biologique et la Théorie de l'information*, Hermann,

[10] A. Turing (1952), "The chemical basis of morphogenesis", *Philosophical Transactions of the Royal Society of London. Series B, Biological Sciences*, Vol. 237, No. 641, pp. 37-72. See also J. Petitot (2013) "Complexity and self-organization in Turing", *The Legacy of A.M. Turing*, (E. Agazzi, ed.), Franco Angeli, Milano, 149-182. ArXiv: http://arxiv.org/abs/1502.05328v1. A. Lesne, P. Bourgine (eds.) (2006). *Morphogenèse. L'origine des formes*. Belin, Paris. Murray J.D. (2005) *Mathematical Biology*, Springer, New York.

I had the privilege to take part in the historical (and polemic) meeting between Jean Piaget and Noam Chomsky organized in 1975 at the Center of Royaumont by Massimo Piattelli-Palmarini, where I presented the principal differences between Chomsky and Thom[11].

Q. How did Thom become interested in linguistics?

Thom much admired Lucien Tesnière. He deeply regretted that he did not meet this great linguist in Strasbourg when he was a young researcher working there with Henri Cartan between 1947 and 1951. His interest focused on the way Tesnière conceived the dependence relations between the constituents of a sentence (still the mereological problem!) and developed an almost narratologic idea of the sentence as a "scene" making actants interact: "The verbal node [...] expresses a small drama by itself. "[12]

Thom was philosophically a realist in linguistics. He estimated that, below the great variability and complexity of the morphosyntaxic surface structures, the universals of language result from evolution and are rooted in the cognitive abilities of primates, in particular in perception and action. Consequently, he tackled the linguistic problems from the point of view of the biological evolution of cognitive structures.

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[11] Petitot, J. (1979) "Hypothèse Localiste et Théorie des Catastrophes. Note sur le Débat", *Théories du Langage, Théories de l'Apprentissage, le Débat Chomsky / Piaget*, (M. Piattelli-Palmarini, ed.), 516-524, Le Seuil, Paris.

[12] L. Tesnière (1959), *Éléments de syntaxe structurale*, Klincksieck, Paris, 1959 (2ème éd. 1988), 48, 1. Voir aussi J. Petitot (1985) *Morphogenèse du Sens. Pour un Schématisme de la Structure*, PUF, Paris.

Q. Which other researchers besides you were interested in Thom's work on linguistics?

Among the linguists and semioticians who very early were deeply interested in Thom, one can quote, besides masters like Jakobson, Seiler and Pottier[13], two researchers of my generation: Wolfgang Wildgen[14] of the University of Bremen in Germany and Per Aage Brandt[15] of the University of Aarhus in Denmark. Their work developed, like mine, in the 1970-80s.

Then, in a completely independent way, without any reference to the European debate, something relatively similar happened in the United States in the 1980-90s with the emergence of West Coast cognitive linguistics: Charles Fillmore and George Lakoff at Berkeley, Len Talmy at Berkeley then at Buffalo, Ron Langacker at San Diego (where Gilles Fauconnier was, too). These linguists developed approaches which, on the one hand, were very structural (although with few references to European structuralism) and, on the other hand, explicitly supported the same theses on the evolutionary origin of language in relation to perception and action. At the time of the conference on Tesnière organized in 1992 in Rouen by Françoise Madray-Lesigne (Tesnière was born in 1893 in Mount-Saint-Aignan close to Rouen), Langacker made an emphatic praise of Tesnière by regarding him as one of his precursors[16] .

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[13] See for example B. Pottier (2000) *Représentations mentales et catégorisations linguistiques*, Paris, Louvain, Peeters.

[14] cf. W. Wildgen (1982) *Catastrophe Theoretic Semantics. An Elaboration and Application of René Thom's Theory*, Benjamins, Amsterdam, and (1999) *De la grammaire au discours. Une approche morphodynamique*, Peter Lang, Bern.

[15] cf. Per Aage Brandt (1994) *Dynamiques du sens*, Aarhus University Press, Aarhus, and (1995) *Morphologies of Meaning*, Aarhus University Press, Aarhus.

[16] cf. Langacker, R.W. (1995) "Structural Syntax: The View from Cognitive Grammar ", *Lucien Tesnière aujourd'hui,* (F. Madray-Lesigne and J. Richard-Zappella, eds.), Actes du Colloque international CNRS, Université de Rouen 16-18 novembre 1992, Louvain/Paris, 13-39.

In addition, connectionist models of neural networks, opposed to formal models, were developing powerfully and rapidly. One of the linguists more in sight in that field was Paul Smolensky whose models raised a violent debate with Jerry Fodor and Zenon Pylyshyn. I took part in that debate[17].

Wildgen, Brandt and myself made contact with these new trends. We organized meetings, for example in San Marino, at Umberto Eco and Patrizia Violi's "International Center for Semiotic and Cognitive Studies", a conference with Len Talmy, and also two important conferences at the CREA on the issue of constituent structures in connectionist models. These conferences were relayed by another one, organized this time at Bloomington by Tim van Gelder and Bob Port, entitled "Mind as Motion"[18].

Q. How were you informed of the works existing in the United States?

I was much interested in Fillmore — Case linguistics, with Tesnière's structural syntax, is the closest to Thom's views —, Langacker, Jackendoff, Lakoff. But the one that made contact with the most interesting linguist for us, namely Len Talmy, was Per Aage Brandt. Len achieved a splendid work on linguistic Gestalt while showing empirically, on a large corpus of data, the

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[17] See for example P. Smolensky (1988). "On the Proper Treatment of Connectionism", *The Behavioral and Brain Sciences*, 11, (1988), 1-74. J. Fodor, Z. Pylyshyn (1988) "Connectionism and cognitive architecture: a critical analysis", *Cognition*, 28, 1-2 (1988) 3-71. J. Petitot (2011) *Cognitive Morphodynamics. Dynamical Morphological Models of Constituency in Perception and Syntax* (with R. Doursat), Peter Lang, Bern.

[18] T. van Gelder, R. Port (eds.) 1995, *Mind as Motion*, Cambridge, MIT Press.

existence of very close connections between syntax (more precisely deep "syncategorematic" structures) perception and action. He went much further than case markers like prepositions[19].

Q. Which modelling for linguistics?

Once the empirical regularities are described, one asks how to model them. I regarded the works by Fillmore, Langacker, Talmy, etc. as well supported results; I trusted them, and what interested me was to see how the syntactico-semantic structures they had identified could be modelled.

For this purpose, I applied a general methodological principle. It is not because the structures under scrutiny are of a linguistic nature that the good tool, *a priori*, is formal languages. In sciences, one should not make any assumption on the fact that the mathematical tools should be of the same order as the objects. I am not anti-formalist *a priori*. I am ready to admit that formal models can prove to be the best in some cases. But I do not see any *a priori* reason that formal languages should constitute good tools to understand syntactic structures of natural languages, no more than to understand perceptive, biological or molecular mereological structures. My methodological principle is: the structures of natural languages are natural phenomena (I underline "natural") and, as in other sciences, it is necessary to invent (I underline "to invent"), starting from appropriate bases, the suitable mathematical tools to model them.

These appropriate bases are two-fold. On the one hand, they come from properly linguistic studies and on the other hand from other disciplines like cognitive sciences and neurosciences. Of

[19] Len Talmy (2000). *Toward a Cognitive Semantics*, Vol. I: *Concept Structuring Systems*, Vol. II: *Typology and Process in Concept Structuring*, Cambridge, MA, MIT Press, 2000.

course, no need to make brain imagery to explain the conditional mood in French, but one must use neurocognitive results on universal sensorimotor schemes of interaction between actants to understand verbal valence.

III. Phonetics, phonology and catastrophe theory

Q. You were interested in phonetics.

In certain cases, non-formal mathematical models of a topological-geometrical-dynamical type have proved to be rather good. The first example which convinced me of the validity of morphodynamical models in the field of language was phonetics. As you know, structuralism comes mainly from the phonological work of the Moscow and Prague Circles, and, when I began to work with Thom, I already knew structural phonology a little and the remarkable results that Jakobson transfered to general structuralism, in particular in collaboration with Claude Lévi-Strauss. I thus tried to test Thom's models on it and I discovered (I consider that it was my first scientific "discovery") that they were completely adapted to phonology.

It happens that at the EHESS there was (and there still is) a very good laboratory of cognitive psychology[20] some members of which worked in phonetics. Following their advice, I read many things in this field and I noted that Thom's models were not only relevant but that they were quantitatively and qualitatively exact.

[20] The LSCP (Laboratoire de Sciences Cognitives et Psycholinguistique).

In phonetic perception, several levels should be distinguished: the acoustic level, the peripheral level of sensory transduction, the perceptive level and the linguistic level. At one end of the chain, one can make a lot of acoustico-physical experiments and at the other end one has at one's disposal a very important linguistic corpus of thousands of languages.

One characteristic of phonetic perception is what is called its "categorical" character. What does that means? When one makes a sonogram one can identify the "formants" of the sounds produced by the vocal tract. Sounds produced by vocal cords are very rich in harmonics, and the articulatory controls control the shape of the resonators of the vocal tract. Each of these resonators (mouth, nose …) amplify or damp specific harmonics. In other words, the amplitudes of the harmonics are modulated by a continuous curve having strongly marked peaks. These resonance peaks select frequency bands which are called formants. Vowels are stationary sounds having characteristic formants, and consonants are transient sounds carrying out transitions between formants and possibly introducing turbulence (plosives, fricatives).

When you look at the equations, you observe that the formants correspond to the maxima of what is called "the transfer function" (the output/input ratio) of the vocal tract. In fact this function H is the inverse of a function G and the maxima of H correspond to the minima of G.

One can simplify the problem by preserving only a few resonators, for example three: the front cavity (mouth), the back cavity (pharynx) and the nasal cavity. Each cavity is described by a tube (with length and diameter) and constrictions of the vocal tract are described by small intermediate tubes. One knows how to explicitly compute the way in which the formants depend on these articulatory parameters. Personnally, I used as a guide the classic *Preliminaries to*

Speech Analysis by Jakobson, Fant and Halle[21]. From this audio-acoustic base, structuralist works, in particular Jakobson's, show how phonological (linguistically relevant) distinctive features can be recovered using a qualitative description of the formant configurations. For example if one considers the universal vocalic triangle /*a*/, /*i*/, /*u*/ in simple models with two formants:

/*a*/ corresponds to close formants of medium frequency (feature "compact"),

/*i*/ corresponds to well separated formants (feature "diffuse") with predominance of the "acute" formant (high frequencies),

/*u*/ corresponds to well separated formants (feature "diffuse") with predominance of the "bass" formant (low frequencies).

If more detailed models are used, one can still qualitatively describe phonological distinctive features in this way by using not the true formants quantitatively defined, but "formantial masses" as Ludmilla Chistovich proposed a long time ago.

Then, I looked at the explicit formulas connecting the formants to articulatory controls and I discovered that, for the models with tubes, the function *G* exactly is an unfolding of singularity in Thom's sense and that the formants and their configurations are consequently describable in terms of catastrophes: there are abrupt — discontinuous — changes in formantial masses according to continuous changes in articulatory controls[22].

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[21] Jakobson, R., Fant, G. & Halle, M. (1952) *Preliminaries to Speech Analysis. The distinctive features and their correlates*, MIT Technical Report.

[22] See J. Petitot (1985) *Les Catastrophes de la Parole. De Roman Jakobson à René Thom*, Maloine, Paris; and (1997) "Modèles morphodynamiques de catégorisations phonétiques", *The Roman Jakobson Centennial Symposium* (P.A. Brandt, F. Gregersen eds), *Acta Linguistica Hafniensia*,29,239-269. http://jeanpetitot.com/ArticlesPDF/Petitot_Jakobson.pdf

Let us be a little more technical. The transfer function is a function *H(s)* of a complex variable $s = \sigma + i\omega$ where $\omega/2\pi$ is the frequency and σ a damping factor. The restriction of $H(s)$ to the imaginary axis ω gives the modulation of the harmonics frequencies. Let us consider the model with one resonator (i.e. two tubes and one formant) of figure 2.

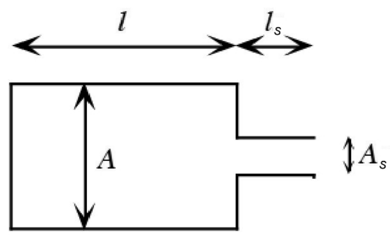

Figure 2. Model with two tubes

One obtains (for conveniently chosen values of l, l_s and A_s) the already complex formula:

$$H(s) = \frac{1}{LC\, s^2 + (RC + GL)\, s + GR + 1} = \frac{{\omega_0}^2}{(s - s_1)\,(s - s_2)}$$

with

$$L = {\rho l_s}/{A}$$
$$C = {lA}/{\rho c^2}$$
$$R = \frac{S}{A^2}\sqrt{\frac{\omega\rho\mu}{2}}(l + l_s)$$
$$G = S\frac{\eta - 1}{\rho c^2}\sqrt{\frac{\lambda\omega}{2c_p\rho}}(l + l_s)$$
,

where A = section of the open tube, S = circumference of diameter A, ρ = density of the air, c =

speed of sound, $\mu =$ coefficient of viscosity, $\lambda =$ coefficient of conduction of the heat, $\eta =$ adiabatic constant, c_p = specific heat of air under constant pressure. The poles of *H(s)* are

$$s_1 = -\frac{1}{2}\left(\frac{R}{L}+\frac{G}{C}\right)+i\sqrt{\frac{GR+1}{LC}-\frac{1}{4}\left(\frac{R}{L}+\frac{G}{C}\right)^2}$$
$$s_1 = \sigma_1 + i\omega_1$$
$$s_2 = \bar{s}_1 = \sigma_1 - i\omega_1$$

$$\omega_1 = \sqrt{\omega_{01}{}^2 - \sigma_1{}^2}\ , \quad \omega_{01}{}^2 = \frac{GR+1}{LC}$$

Only s_1 is relevant because its imaginary part is > 0 and frequency must be > 0.

For a model with two resonators (four tubes and two formants), one obtains the formula :

$$H(s) = \frac{\omega_1\,\omega_2}{s^4 + as^3 + bs^2 + cs + d}$$

with

$$\omega_1 = \frac{1}{\sqrt{L_1 C_1}} \ , \ \omega_2 = \frac{1}{\sqrt{L_2 C_2}}$$

$$a = \frac{R_1}{L_1} + \frac{R_2}{L_2} + \frac{G_1}{C_1} + \frac{G_2}{C_2}$$

$$b = \frac{1 + R_1 G_1}{L_1 C_1} + \frac{1 + R_2 G_2}{L_2 C_2} + \frac{R_1 G_2}{L_1 C_2} + \frac{R_2 G_1}{L_2 C_1} + \frac{G_1 G_2}{C_1 C_2} + \frac{R_1 R_2}{L_1 L_2} + \frac{1}{L_2 C_1}$$

$$c = \frac{\left(R_1 (1 + R_2 G_2)\right)}{L_1 L_2 C_2} + \frac{\left(G_2 (1 + R_1 G_1)\right)}{L_1 C_1 C_2} + \frac{R_1 + R_2 + R_1 R_2 G_1}{L_1 L_2 C_1} + \frac{G_1 + G_2 + G_1 G_2 R_2}{L_2 C_1 C_2}$$

$$d = \frac{\left(1 + R_1 G_1 + R_2 G_2 + R_1 G_2 + R_1 R_2 G_1 G_2\right)}{L_1 C_1 L_2 C_2}$$

Figure 3 shows the graph of the logarithm of the module of $H(s)$ and the damping of the formants.

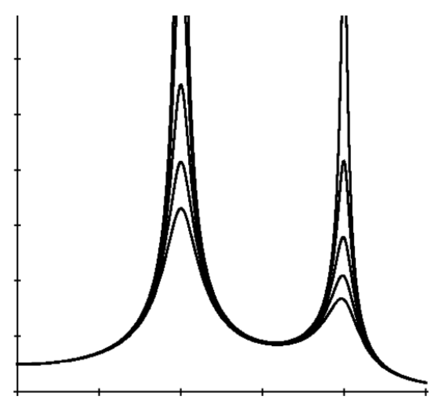

Figure 3. Damping of the two formants.

The key point is that, for *n* formants, (*i*) the denominator of the transfer function *H*(*s*) is the universal unfolding of the singularity known as A^{2n}

$$A^{2n}(s) = s^{2n} + a_{2n-1}\, s^{2n-1} + \quad + a_1\, s + a_0$$

and (*ii*) the coefficients of this unfolding are complicated functions of the 2*n* articulatory controls.

To move from formants to formantial masses, and thus from quantitative to qualitative, one introduces an "auditive transformation" which merges the sufficiently close formants. Then one really obtains models in the sense of Thom.

In short, one can, thanks to Thom's models, explicitly move from the acoustic level (physical) to the auditive level (sensorial) then to the phonological level (linguistic), the key being the interpretation of formantial masses in terms of unfoldings of singularities parameterized by articulatory controls.

Let us address now the issue of categorical perception[23]. In addition to articulatory controls determining the shape of the vocal tract, there exist other acoustic cues that one can vary in a continuous manner, e.g. voicing (VOT: voice onset time) that measures the moment of excitation of the fundamental harmonic. In short, while one can vary many parameters continuously, perception does not vary continuously. It is the fundamental reason why some sounds can be the substrate of a phonological code. For example, you can vary voicing in order to move from [b]

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[23] Among a rich bibliography, three references were important for me: Liberman, A. M., Cooper, F. S., Shankweiler, D. P., Studdert-Kennedy, M., (1967) Perception of the Speech Code, *Psychological Review*, 74, 6, 431-461. Stevens, K., (1972) The Quantal Nature of Speech, *Human Communication, a Unified View* (P. B. Denes, E. E. David Jr. eds.). Malmberg, B., (1974) *Manuel de phonétique générale,* Paris, Picard.

(voiced labial) to [p] (not-voiced labial). But at the perceptive level, on the other hand, you perceive only allophones of /*b*/ or /*p*/ and no intermediate state.

To explain this remarkable phenomenon, psychologists distinguish two fundamental mechanisms. On the one hand, discrimination: can I discriminate two close [b]; and on the other hand, identification: do I identify a /*b*/ or a /*p*/, i.e. a sound as an allophone of a phoneme or of another one.

For colours, discrimination corresponds to shades and identification corresponds to colour names. The perception of colours is "continuous" in the sense that shade discrimination depends very little on colour identification: one perceives gradual shades independently of the existence of the categories of colours. In categorical perception, the situation is quite different: discrimination degenerates inside the categories; as one says, it is subordinated to identification: one discriminates two close sounds only if they are identified as different. One is unable to discriminate two close [b] sounds identified as allophones of /*b*/, but on the other hand one is able to discriminate two close intermediate sounds if one is identified as an allophone of /*b*/ and the other as an allophone of /*p*/.

It is a little as in geography: there are areas delimited by boundaries (the domains of the parameter space corresponding to a single phoneme), inside an area the various positions (allophones) have an equivalent type (they are tokens of the same phoneme: no intra categorial discrimination), but at the boundary crossing, the type (the corresponding phoneme) changes abruptly. In categorical perception, there exist thresholds between categories which are induced by perception itself. That is due to the fact that percepts vary in an extremely non-linear way compared to their audio-acoustic and articulatory controls. Kenneth Stevens well studied this phenomenon in his article quoted above "On the quantal nature of speech".

Categorical perception is a fundamental property of phonetic perception and, I repeat, explains why and how some sounds can become the substrate of a code.

Catastrophe theory is particularly well adapted to the modelling of categorical perception because its general model rests on the concept of “bifurcation”. A bifurcation occurs in a system when a small change of a continuous control produces a qualitative jump of the internal state of the system, in other words when a small variation of causes involves great differences on effects. It is precisely what occurs with categorical perception when, for example, a small articulatory change qualitatively moves the configuration of formantial masses from a single formantial mass to two formantial masses (“compact/ diffuse” opposition).

I thus showed that the catastrophes related to audio-acoustic equations match the phonological structures observed in languages. For phonetics, Thom’s models are thus valid models. Jean-Luc Schwartz and the Grenoble group, among them Christian Abry and Louis-Jean Boë, went more thoroughly into them[24]. In particular they identified the “auditive transform” as a mechanism of “large scale spectral integration”.

To summarize, catastrophe models help understanding in a detailed way the link between audio-acoustics, psycho-physics, perception and structural phonology.

Q. What exactly is a catastrophe model?

A catastrophe model starts with a system which has internal states. For instance, in the case of phonetic perception, there are neuronal states corresponding to percepts. In the case of a

[24] See for example Abry, C., Boë, L.-J., Schwartz, J.-L. (1989). Plateaux, catastrophes and the structuring of vowel systems. *Journal of Phonetics* 17, 47-54. Schwartz, J.-L., Boë, L.-J., Vallée, N., Abry C. (1997). The dispersion-focalization theory of vowel systems. *Journal of Phonetics* 25, 255-286.

chimical element such as water, the thermodynamical states are called “phases”: solid, liquid, gas. These internal states are attractors of the internal dynamics of the system and the transient states, induced by the inputs of the system, are rapidly stabilizing toward them: for instance, an acoustic input turns into a perceptive state (after having gone through the external ear, the cochlea, the auditory cortex). Moreover, the system is controlled by external parameters (articulatory parameters, acoustic cues, temperature, pressure…). When these controls change, the inner states change in turn, and there is two possible outcomes. Either a small change in the controls is without consequences and does not change the inner state qualitatively (e.g. the sound is still perceived as a /*b*/, the water temperature shifts from 50°C to 51°C), or a small change in the controls changes the qualitative type of the inner state (the sound is now perceived as a /*p*/, the water temperature shifts from 99°C to 100°C and the water starts boiling). Such qualitative changes are called “bifurcations”. In thermodynamics they are called “phase transitions”.

Q. Can a shift from /*b*/ to /*p*/ be predicted, in the sense that it follows some constraints?

There are strong differences across languages. But one can assume a universal innate “initial state” for new born humans. During language acquisition, some thresholds move, others split, others disappear. For instance, in the case of the Japanese language, the threshold between [r] and [l] disappears and [r] and [l] become two allophones of a single phoneme. Young Japanese are able to discriminate between [r] and [l] but, while learning the language, these discriminations disappear due to the categorical property of perception.

Thus, maybe there is an initial phonological “geography” that evolves with the learning of a specific phonological system. All the phonological systems categorize the same space of sounds defined by anatomically possible articulatory controls and harmonics. The question is to know whether the categorisations of actual languages are efficient and whether they reach an optimum of the quantity of information conveyed by the phonological code.

Q. Is there a definition of what is an optimum vocalic system for communication?

This is a fascinating question. We know of a great number of phonological systems and they may be grouped into classes. Numerous models have been proposed. The problem is the following: within the space of the possible sounds, defined by universal anatomic constraints, we have to find the best categorisation into sub-regions (the phonemes). There are several strategies in order to solve the problem of the optimisation of the categorisation and there are several studies showing how these strategies are related to each other. All the phonological systems are based on the universal vocalic triangle /*a*/, /*i*/, /*u*/, and can be described as a progressive refinement complexifying that triangle, leading eventually to the most complicated vocalic systems, such as that of French.

Q. We mentioned "basins of attraction". Could you explain that notion more thoroughly?

When you have a dynamics defined on a given space M, all the points x belonging to M have a trajectory $\gamma(x)$ and you can consider the asymptotic behaviour of that trajectory. Generally, $\gamma(x)$ is attracted by an attractor A which is a sub-set topologically closed and dynamically invariant, minimal for these two properties, and which attracts all the trajectories coming from points in its neighbourhood. All the points x whose trajectory $\gamma(x)$ are asymptotically attracted by A constitute the basin of attraction $B(A)$ of A. Thus the dynamics decomposes M in several basins of attraction separated by boundaries (some of them can be very complicated).

In the case of the internal dynamics of a system, every input puts the system into an initial state x and, generally, x is not on the boundary but inside a basin of attraction $B(A)$ and is therefore attracted by the attractor A. This means that the initial transient state x will be attracted by the internal state A. This projection of the input on an attractor models the process of "identification". In this type of models for categorisation, the basins of attraction are the categories and the attractors are the prototypes. An input induces an initial state that is associated with a prototype. In the phonetic domain, a sound is recognized and identified as an allophone of a given phoneme.

Some boundaries between basins of attraction are more complex than others. When an initial state is on a boundary separating two basins as a sort of ridge-line, it is possible to fall into one or the other of the basins. And, last but not least, the control space allows the modification of the basins of attraction and their boundaries.

That is why Thom proposed to distinguish two kinds of bifurcation: (*i*) internal bifurcations, when the system moves from a basin of attraction to another in the internal space *M* and (*ii*) external bifurcations, when the system is coerced into another attractor by the effects of the controls, for instance due to the fact that an attractor disappeared or that two attractors have merged. In practice, external bifurcations matter most: the systems are in general of the "slow/fast" kind, which means that the internal dynamic is fast, while the variation of the controls is slow and then it is possible to do as if the system were always in an internal stable state (on an attractor). It is called an "adiabaticity hypothesis". What chiefly matters then is the bifurcation of the attractors and not the fast internal transient trajectories.

Q. From a mathematical point of view, which kinds of mathematics are involved?

When building the mathematics for his models, Thom chose – for the spaces, the functions and the maps between spaces he needed – the level called differentiable, that is the level where the objects have locally almost everywhere well-defined derivatives, except sometimes in some singular points. This level is more constrained than the basic continuous level (he does not allow objects such as fractals). However, it is far less constrained than the algebraic or metric level. The differentiable objects are very "flexible".

Thom introduced two kinds of models: the elementary models, and the extended ones. In the first ones, the internal dynamics is the steepest descent of an energy potential function $f(x)$ defined in the inner space *M*: the system optimizes its state by minimizing its internal energy.

The attractors (the internal states) are in that case the minima of $f(x)$: an initial state goes (according to the specificities of the system) either to the closest minimum or to the absolute minimum.

The control parameters allow the variation of the potential functions, and therefore the change of the minima and their height. A minimum may then disappear and the system will have to go into another minimum: these are bifurcations.

One of Thom great achievements has been the (difficult) proof of the classification theorem for elementary catastrophes. The main idea is the following: if you consider a potential function f where several minima, maxima or saddles (called "critical points") are merged in a single point x, f has an unstable singularity at x (according to a natural notion of stability). If you deform such a singularity through external small parameters w embedding f into families $f_w(x)$ defined in a small neighbourhood of x with $f_0(x) = f(x)$, it is possible to stabilize the singularity in many ways, partially or totally, through the dissociation of the critical points that have been merged. The key result is that, given such a singularity, there exists a universal deformation, called "universal unfolding", that gathers optimally all the possible stabilisations.

Figure 4 shows the catastrophe named "cusp", that plays the key role in modelling the universal vocalic triangle. The unstable singularity x^4 merges two simple minima (non-degenerated minima, i.e. that are not composed by merging simpler critical points) and one simple maximum. The external space W of the universal unfolding is two dimensional. It is partitioned into three regions by a catastrophe set K, containing the two branches K_b of a cusp curve and the median half-line K_c. Along the branches K_b one simple minimum remains simple while the other simple minimum and the simple maximum merge into an inflection point: the K_b are lines of catastrophes of bifurcation. Along K_c, the two simple minima and the simple maximum remain simple but the two minima have now the same height: K_c is a line of catastrophes of conflict. Apart from K, $f_w(x)$ have either a simple minimum, either two simple minima separated by the simple maximum with one of the minima that dominates the other.

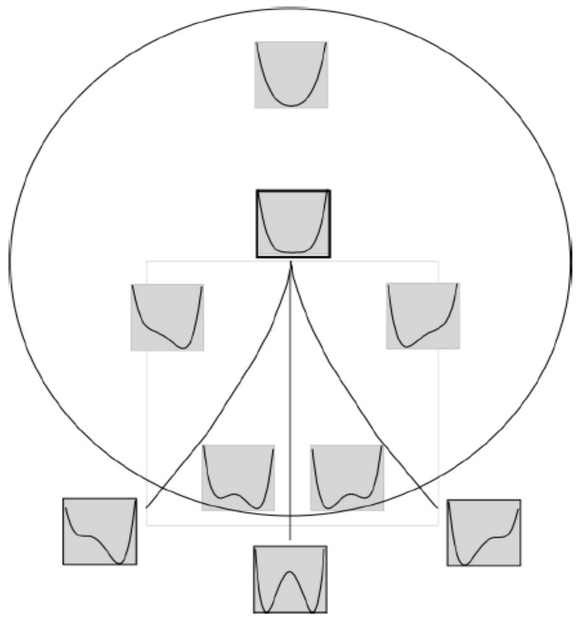

Figure 4. The universal unfolding of the “cusp” catastrophe

The classification theorem says that, whatever the system under scrutiny, if there are one or two internal dimensions and no more than four external controls w, and if the potential function $f(x)$ of the system has unstable singularities, and if its unfolding is structurally stable (that is the process stabilizing internal unstabilities is itself stable), then these singularities belong to a finite list: “cusp”, “swallowtail”, “butterfly” in the one dimensional case, elliptic, hyperbolic or parabolic “umbilic” in the two dimensional case.

From a methodological point of view, this result is very important because it exhausts the field of possibilities. It is as important as the theorem of classification for the platonic solids (the finite sub-groups of the group of the rotations) [25].

[25] Readers interested in the theory of mathematics may wish to read (in French) : Chenciner, A., (1980). “Singularités des fonctions différentiables”, *Encyclopædia Universalis*, Paris ; as well as the

III. Language and perception

Q. What are the consequences for linguistics?

As I said Thom was interested in a realist approach to language. For him, language had an evolutionary origin. The ability to describe perceptive scenes of the outer world communicate them to those that do not see them was, for him, a fundamental requisite constraining natural languages. It appears that a large part of these perceptive scenes are interactions between "actants"[26] (being either agents or objects), and the transformations of their spatial relations can be described by verbs (to go into a place, to seize something, to attack, to run away, etc.).

The basic assumption is that the structure of the sentences describing a perceptive scene with verbs and actants situated in space and time is a result of the evolution pressure and that there is an analogy between the constituent structure (mereology) of actantial syntax and the constituent structure of the perceptive scenes.

This assumption of a foundation of the actantial structures in the structures of perception and action has a long history. One of its components is the "localist hypothesis", which has been supported under various guises by linguists such as Anderson, Langacker or Talmy, and according to which the basic syntactic structures of elementary sentences categorize the generic interactions in space and time. Structural syntaxes like Tesnière's and case grammars like Fillmore's belong to the same paradigm. All these theories rely on an actantial theory using

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compilation I made in 1982 (in French): *Eléments de théorie des singularités*, http://jeanpetitot.com/ArticlesPDF/Petitot_Sing.pdf

[26] We use the term "actant" analogue to what are called "semantic roles" in case grammars. It is a deeper concept than that of "actor" or "character".

semantic roles defined by spatio-temporal schemas similar to schemas of perception and action and therefore rooted in cognitive evolution.

In this context, Thom's theorem is of primary importance: it is possible to classify the actantial spatio-temporal interactions thanks to the classification of elementary catastrophes. This theorem proves the existence of case universals. It is obviously a fundamental result.

These hypotheses have been controversial. Numerous linguists objected that language is independent from perception and that it is a cognitive faculty *sui generis*. Chomsky for instance argues for the notion of autonomy of syntax. Other linguists acknowledge the existence of links with perception but argue that the linguistic categories of perception cannot be extracted from the perception itself. And, in any case, these hypotheses are but of low interest for linguists describing actual natural languages since they pertain to a deep "proto-linguistic" level, far below from the morphosyntactic diversity of natural languages. However, these hypotheses have a great theoretical significance for building bridges between linguistics and cognitive neurosciences. They have also a great technological relevance, in order to build robots able to convert natural language instructions in terms of perceptual structures and motor programs.

Q. Could you elaborate upon the relation between localist hypothesis and catastrophe theory?

If one try to schematize perceptual scenes and actantial relations (schematization is a strong simplification focusing only on the essential forms), one encounters again structures that are derived from elementary catastrophe.

Let's take objects distributed in space, that is to say static configurations of spatial actants. Let us add a temporal evolution that changes this configuration dynamically. These temporal evolutions generally lead the actants to interact. It is then possible to represent the actants through minima of a potential function (here is the schematization) so that to turn the interactions into bifurcations and so that it is possible to apply the models of elementary catastrophes. I explain this in details in *Cognitive Morphodynamics*[27]. A schema such as "to take an object" is the fact that there is an actant and an object which are initially disjoint and which, later, are conjoint. The verbal node lexicalized by the verb "to take" describes this interaction, which is a bifurcation derivable from the cusp catastrophe. As soon as early 1970s, Thom made the list of the "archetypal actantial graphs" that are derivable from elementary catastrophes[28].

Later on, Thom's archetypes have turned out to be great precursors of several cognitive models of language: Fillmore's frames, Langacker, Talmy, Lakoff's image-schemas, Haiman's "iconicity in syntax", Desclés' "cognitive archetypes", Shank and Abelson's scripts, etc. (for more details see *Cognitive Morphodynamics*[29]).

Using the fact that elementary verbal nodes grammaticalize bifurcations of actantial relations, you can build a theory of verbal valency. It was one of the results that mattered the most for Thom. All the linguists that have been interested in verbal valency know that there is a limit of 4 actants (the few controversial cases with 5 actants use indeed a double actant). Where

[27] Peter Lang, Bern, 2011.

[28] See for instance "Topologie et linguistique", *Essays on topology and related topics*, A. Haefliger and R. Narasimhan (eds), Springer, 1970, 226-248. Reprinted in: *Modèles mathématiques de la morphogenèse*, Paris, 10-18 UGE, 1974.

[29] See for instance Fillmore, C., (1976) "Frame semantics and the nature of language", In *Annals of the New York Academy of Sciences: Conference on the Origin and Development of Language and Speech*. Volume 280, 20-32. Haiman, J., (ed.) (1985) *Iconicity in Syntax*, Amsterdam, J. Benjamins. J.-P. Desclés (1990) *Langages applicatifs, Langues naturelles et Cognition*, Paris, Hermès. Schank, R., Abelson, R.P. (1977) *Scripts, Plans, Goals and Understanding*, Hillsdale, Lawrence Erlbaum.

does this limit come from ? Why could not we be able to create new semantic roles allowing for an increase of the valency ? According to Thom, it is one of the strongest evidences of the rooting of actantial syntax into perception and action. Indeed, perception and action take place in a 4-dimensional space-time, and archetypal actantial graphs derive from elementary catastrophes whose external space have 4 dimensions at most. This closed list of catastrophes, drawn from the classification theorem, puts a drastic limit on the complexity of the bifurcations and, then, on the verbal valency. We then observe that, in all archetypes, the valency has a limit of four[30]. According to Thom, this constraint comes from our outer world.

Of course, several linguists objected that, in most of the verbs denoting action, there is an agentivity, and that agents are generally intentional agents. However, numerous remarkable experiments have shown how strongly agentivity itself is deeply rooted in perception and action. As early as the 1940s, F. Heider and M. Simmel have shown that movements, even complex ones (such as movements including accelerations, decelerations, changes of direction, etc.) of simple forms (triangles, circles, rectangles of various sizes) were spontaneously described by way of intentional action verbs ("come in", "come out", "give", but also "hide", "escape", "hunt", "attack", "force", etc. [31]). Since these pioneering experiments, numerous works were devoted to such phenomena. Let us mention for instance J. Scholl and P. D. Tremoulet on the perception of causality and the animacy of objects; D. Premack on the perception of intentional movements by children; S. J. Blakemore and J. Decety on the comprehension of intentions; M. E. Zibetti on the fact that we interpret as if the movements we perceived were caused by intentional agents [32]. All

[30] With 4 as the number of dimensions d of space-time and with 4 as the maximum valency $v(d)$, we have $v(4) = 4$. But it is not the case that $v(d) = d$ generally.

[31] Heider, F., Simmel, M. (1944) "An experimental study of apparent behavior", *American Journal of Psychology*, 57 (1944) 243-259.

[32] See for example Scholl, B.J., Tremoulet, P.D. (2000) "Perceptual causality and animacy", *Trends in Cognitive Science* 4(8), (2000), 299-309. Blakemore, S. J., Decety, J. (2001)

these works try to unveil the evolutionary and cognitive roots of the tendency we have to interpret purely cinematical and dynamical motions as if they resulted from an intentional agentivity. All these authors showed that this tendency is automatic, non-conceptual, "hardwired" and "rooted in automatic visual processing". Their works offered a general confirmation of Thom's theses.

Q. How to build a model of the perceptive scene?

This is an old and interesting issue. In order to move from perception to language, it is necessary to describe the perceptive scenes very schematically in order to specify the relevant information that should be translated into elementary proto-sentences. But how to simplify the perceptive scenes in a bottom-up and data-driven way, and how to define the linguistically relevant information that lays inside?

For instance, let us consider a preposition as "across" (this example is drawn from my book *Cognitive Morphodynamics*). This preposition can be applied to a huge diversity of perceptive scenes. "Across" can be applied to the crossing of a street, a lake, a field, a country, to walking haphazardly into a forest, etc. How would it be possible to define the geometric and topological invariant content of "across"? Obviously, that invariant pertains to the notion of "transversality", but how could we design algorithms extracting such a schema from a real and complex scene? The problem is to find good tools for simplifying the scene, tools strong enough for skeletonizing it, but also able to preserve the relevant information (" transversality "). In *Cognitive*

"From the perception of the action to the understanding of intention", *Nature Reviews Neuroscience* 2, (2001), 561-567. Zibetti, E., Tijus, C. (2003) "Perceiving Action from Static Images: the Role of Spatial Context", *CONTEXT* (2003) 397-410.

morphodynamics, I showed, together with René Doursat, that some morphological algorithms implemented in neural networks can do the job[33].

Figure 5 schematizes the clause "zigzagging across the woods": (*a*) is the input image; it contains two objects: the path and the wood. (*b*) and (*c*) result from a first preprocessing using morphological algorithms of dilation and skeletonization. (*d*) and (*e*) result from a second preprocessing; in (*e*) the intersection of skeletons *sk*(*A*) and *sk*(*B*) allows one to extract the invariant schema of "transversality".

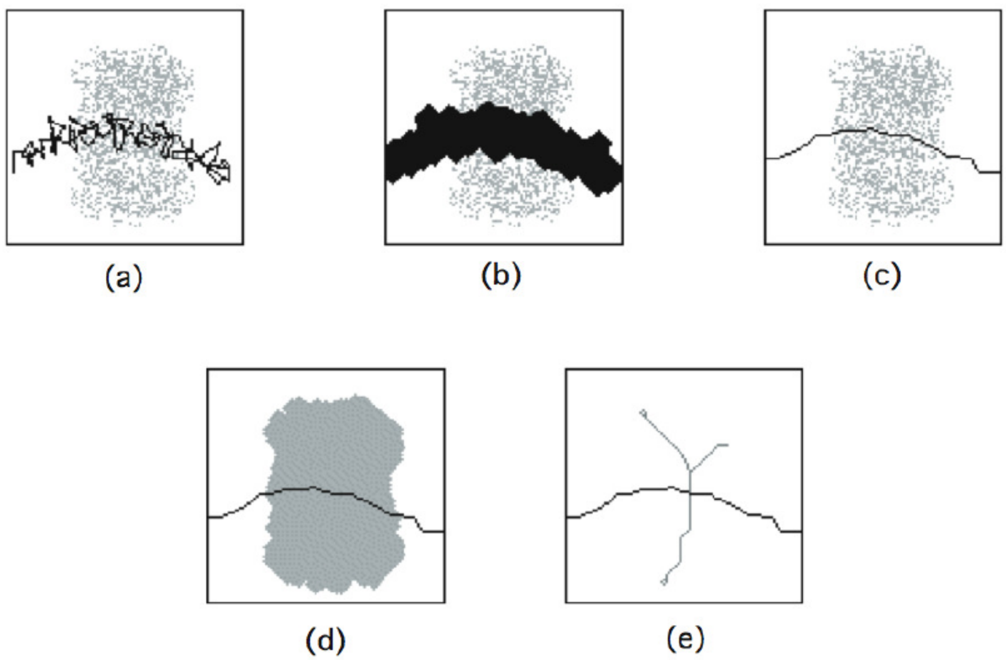

Figure 5. The extraction of the invariant content "transversality" in a pixelized image described using the preposition "across".

[33] These algorithms are discrete variants of dynamic processes of diffusion and skeletonization often used in morphodymics. They are computationally very efficient. They have been developed by G. Matheron and J. Serra and their colleagues. See for instance: Serra, J. (1982). *Image Analysis and Mathematical Morphology*, New York, Academic Press, 1982.

I stress the fact that these morphological analyses of images are not obvious at all and that our capacity to easily and correctly apply prepositions to visual scenes is far from being understood. This issue is far most complex than that, already quite difficult, of hand writing recognition.

IV. Modelling and simulation

Q. What is the relation between modelling and simulation?

Models are mathematical, but do not belong to reality. I am not a realist concerning mathematics. Let us consider classical physics. Newton's equation is perfect, but planets do not do differential calculus. They move, but they do not solve equations. However, Newton's equation allows for computing (either explicitly or only numerically) some solutions that simulate perfectly the observed motions. The same holds for all models. We start with collecting a great corpus of empirical data and then we try to find good models able to generate a virtual reality that simulates the empirical reality.

If the morphodynamical models based on the hypothesis of the rooting of language in perception are correct, then it would be right to try to understand their neuronal implementation. It is not easy at all. Stephen Kosslyn, a well known neurophysiologist of vision, has studied with current methods of brain imaging the neural activity during the use of prepositions. He showed that there exist two systems for the processing of spatial relations: one is a continuous quantitative processing (A is more or less above B), and the other is a categorical discontinuous processing (A is above or beside B). Moreover, the two neural processings are lateralized: the continuous one takes place in the right hemisphere, while the categorical one takes place in the left hemisphere[34]. Hence, if one wants to know exactly how the brain deals with prepositions, he has to go deeply into the analysis and modelling of the link between perceptive structures and linguistic categorization.

Interested readers will find more information in *Cognitive Morphodynamics* as well as in my 2008 book *Neurogéométrie de la vision. Modèles mathématiques et physiques des architectures fonctionnelles*[35] in which I deal with the neural implementation of basic properties of perception (which are already very difficult to understand even though they remain very far from the complexity of language).

Q. Is simulation a form of explanation?

It depends on the structure of the models on which the simulation is based. “It works!” is not by itself an explanation since it can pertain to the mere fine-tuning of ad hoc parameters. Models are explanatory when they arise from general and strong hypotheses while being able at the same time to generate good simulations. It is the case with Newton’s equation, which results from general physical principles; it is the case with the elementary catastrophes which result from general principles of structural stability and from the dimensions of space-time.

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[34] Kosslyn, S.M. (2006). “You can play 20 questions with nature and win: Categorical versus co-ordinate spatial relations as a case study”, *Neuropsychologia*, 44 (2006) 1519-1523. See also Kemmerer, D. (2007) “A Neuroscientific Perspective on the Linguistic Encoding of Categorical Spatial Relations”, *Language, Cognition and Space*, (V. Evans et P. Chilton eds), *Advances in Cognitive Linguistics*, London, Equinox Publishing Co.

35 Les Editions de l'Ecole Polytechnique, Distribution Ellipses, Paris.

Q. Numerous linguistics phenomena are quantitatively characterized by a Zipfian distribution. Is there any relation between this characteristic and the modelling proposed by catastrophe theory?

I have never worked in the field of statistical linguistics. Regarding Zipf's law in particular, I haven't worked on this subject, although the CAMS did work a lot on it[36].

However, one can't ignore that statistics are a good way for approaching regularities and that, during acquisition, children are learning rules in a statistical way: they extract linguistic rules by generalizing over a finite set of examples.

There are very interesting connectionist models (those by Jeffrey Elman seem to me to be the most interesting) that model how the syntagmatic statistical regularities induce semantic paradigms.[37]. You consider a small corpus containing various classes of nouns (animate agent, non animate object, etc.) and various classes of verbs (*to eat, to read*…). Then you do supervised learning with a neural network: you give a word as input to the network, you ask it to add one more word, and you correct it if it outputs an incoherent sentence. At the beginning, the network produces outputs that haven't any coherence, neither syntactically nor semantically. The corrections you pointed-out allow it to change its internal structure (i.e. to change the weight of its hidden layers) by retro-propagating the errors. When learning is done, the network does not make errors anymore. Then, you look at its hidden layer, and you see that it has built paradigms

36 Cf. Micheline Petruszewycz, "L'histoire de la loi d'Estoup-Zipf: documents", *Mathématiques et sciences humaines*, 44 (1973) 41-56.

[37] Cf. Elman, J. (1989). Representation and Structure in Connectionist Models, *Cognitive Models of Speech Processing*, (G. T. M. Altmann, ed.), Cambridge, MA, MIT Press, 1989, 345-382.

(animate agents, inanimate objects, state verbs, transitive and intransitive action verbs, etc.). “Paradigm” here means that the words are grouped into clusters. In other words, in order to produce correct syntagmatic sentences, the network has built some semantic rules.

Q. In the field of complex systems, what are the differences today between dynamical models and connectionnist models?

The difference between (morpho-)dynamical models and connectionist models is the following: connectionist models do have internal dynamics and, hence, attractors. They are made of atomic units (the formal neurons) linked by inhibitory/excitatory connections having synaptic weight. Each unit influences the units to which it is connected, which produces a global internal dynamics of the network.

The main interest of these connectionist models is to make explicit the underlying differential equations, whereas they remained implicit in Thom’s and Zeeman’s works. These equations (introduced by Jack Cowan, Hugh Wilson and John Hopfield[38]) have very interesting properties and look very similar to those found in statistical physics in the theory of spin glasses. This has allowed, during the 1980s, a massive transfer of a large bulk of results from statistical physics to connectionist models.

But the fundamental limit of connectionist models is that they do not model the bifurcations of attractors that can result from an external dynamics modifying the attractors. They do use

[38] H.R. Wilson and J.D. Cowan (1972). Excitatory and inhibitory interactions in localized populations of model neurons, *Biophys. J.*, 12 (1972) 1–24. J. J. Hopfield (1982). Neural networks and physical systems with emergent collective computational abilities. *Proceedings of the National Academy of Sciences of the USA*, 79, 8 (1982) 2554–2558.

external dynamics but chiefly for modelling learning processes. The consequence is that they cannot afford models for constituent structures needed by all syntactic theories. A sharp debate took place at the end of the 1980s between classic cognitivism (Jerry Fodor and Zenon Pylyshyn) and connectionist cognitivism (Paul Smolensky). Fodor and Pylyshyn's thesis was that if one models the components of a sentence by attractors of a neural network, then it is not possible to model constituency. They were right. In order to model syntax, a model needs to be able to model constituency, which is impossible with attractors only.

However, as I wrote[39], Thom's actantial models provided an answer at the beginning of the 1970's, to this key issue of the late 1980's! Indeed, thanks to their built-in bifurcations, these models allow for what I call an "attractor syntax". If one models constituents (for instance, actants) with the attractors of some network, then it is not possible to model the relations between these constituents (for instance, actantial relations in a verbal node) through the attractors of the same network. One needs interactions between attractors, that is bifurcations. Attractors' bifurcations allow for the dynamical modelling of verbal nodes and constituent structures. It was the central idea of Thom's actantial graphs we have already discussed.

[39] (1991) Why Connectionism is such a Good Thing. A Criticism of Fodor's and Pylyshyn's Criticism of Smolensky, *Philosophica*, 47, 1 (1991) 49-79. (1994) Attractor Syntax: Morphodynamics and Cognitive Grammars, *Continuity in Linguistic Semantics*, (C. Fuchs et B. Victorri eds), Amsterdam, John Benjamins, 1994, 167-187. (1995) Morphodynamics and Attractor Syntax. Dynamical and morphological models for constituency in visual perception and cognitive grammar, *Mind as Motion*, (R. Port and T. van Gelder eds.), Cambridge, MA, MIT Press, 1995, 227-281. Articles summarized in *Cognitive Morphodynamics*.

Q. Have these models proved seminal? What are the actual research results that are based on your work in cognitive morphodynamics?

We already talked about phonetics. In actantial syntax, the most important works are those by my friends Wolfgang Wildgen and Per Aage Brandt. In the teams of Aarhus and Copenhagen Peer Bundgaard[40], Svend Østergaard and Frederik Stjernfelt have used morphodynamic models. In Paris, David Piotrowski, a structuralist in the line of Hjelmslev has elaborated upon my propositions and plans to use neuroimaging (EEG). He claims that good neuroimaging experiments may help decide between linguistic theories since acceptability may be tested with neural waves, in particular N400[41].

Again in the field of linguistics, there are works by Bernard Victorri on synonymy that use dynamic models in an innovative way[42]. About prepositions, there are many works that still need to be modelled, in particular those by Claude Vandeloise[43]. In the volume edited in tribute to Vandeloise, there is a very interesting paper by Langacker[44].

In the field of perception, perceptive bifurcations have been studied extensively. There are models that follow Thom explicitly, others that follow Prigogine, and others synergetics; however, all these models are based on bifurcations. There is a large amount of empirical data.

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[40] Cf. for instance P. Bundgaard and J. Petitot (eds), (2010) *Aesthetic Cognition*, Special Issue of *Cognitive Semiotics*, 5, 2010. F. Stjernfelt and P. Bundgaard (eds) (2011) *Semiotics. Critical Concepts in Language Studies*, New York, Routledge.

[41] Piotrowski, D. (2009) *Phénoménalité* et *objectivité linguistiques*, Champion, Paris.

[42] See Victorri B., Fuchs C. (1996), *La polysémie. Construction dynamique du sens*. Paris, Hermès.

[43] Vandeloise, C. (1986) *L'Espace en Français: Sémantique des prépositions spatiales*, Paris, Editions du Seuil. (2009) "The genesis of spatial terms", *Language, Cognition and Space: the State of the Art and New Directions*, (V. Evans, P. Chilton eds), London, Equinox (*Advances in Cognitive Linguistics*), 157-178.

[44] Langacker, R. (2010) Reflections on the Functional Characterization of Spatial Prepositions, *Espace, Préposition, Cognition. Hommage à Claude Vandeloise*, (G. Col, C. Collin, eds), Corela.

For instance, the Necker cube (figure 6), with its well-known double perspective. The same bi-dimensional stimulus can be interpreted as a tri-dimensional object in two different ways, and these two ways are bifurcating one in the other in a spontaneous and alternating manner along temporal series that have been studied in depth. The inversion of perspective is easy to understand. In bi-dimensional images, there are two points particularly salient and informative (the two edges in the centre of the figure); and according to the way you focus on one or the other of these two points, the cube is seen under one or the other perspective. There is also the example of the Rubin's face (figure 7) [45].

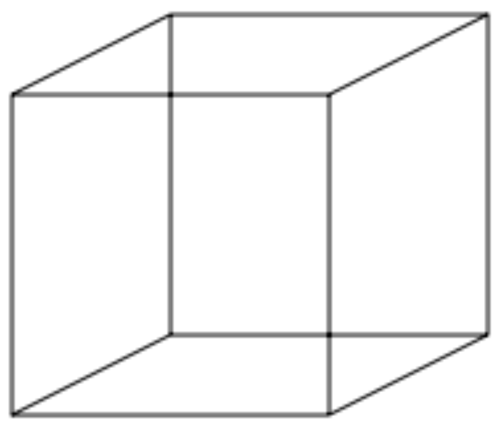

Figure 6. Necker's cube

(source: http://fr.wikipedia.org/wiki/Cube_de_Necker)

[45] The vase-face by Edgar Rubin (Rubin, 1921) shows the importance of the figure–ground contrast in perception. According to whether one looks at the white area as the ground or the form, one sees two faces or a vase.

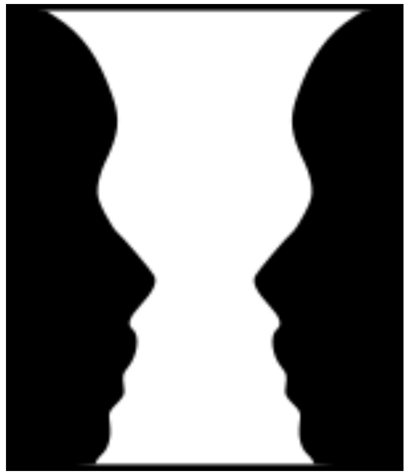

Figure 7. Rubin’s face

(source: http://fr.wikipedia.org/wiki/Perception_figure-fond)

Thus, in many domains where mereological concepts of morphology and of structure mean something, one of the major issues is understanding how categories can emerge in continuous substrates. For this, one needs models where, in one way or another, there exist processes that produce discontinuities. One cannot escape this necessity and this explains the relevance of morphodynamical models.