Acknowledgements

NEW REMARKS ON THE MORPHODYNAMICAL SCHEMATIZATION OF THE CANONICAL FORMULA FOR MYTHS

Jean Petitot
EHESS, Paris
petitot@poly.polytechnique.fr

To Claude Lévis-Strauss
with deep admiration

This new reflection on the canonical formula for myths (*CF*) proposed in 1955 by Claude Lévi-Strauss provides me with the opportunity to clarify the morphodynamical model I proposed in 1988 in the journal *L'Homme (H1)*[1]. During the exciting seminar organized by Professor Solomon Marcus at the Collège de France de Paris in the fall of 1993, I already clarified some of its epistemological difficulties. This work was published in *L'Homme* in 1995 *(H2)*[2]. I will attempt here to explain in further detail the so-called "double cusp" model presented in *H1*. My basis will be the pioneering work of Pierre Maranda[3] and Lucien Scubla's outstanding Dissertation[4],

Since this paper must remain within reasonable limits, I cannot, of course, present either the history of the *CF* or the details of morphodynamical modeling. For the former, I refer the reader to Scubla's treatise, now the authoritative reference on the subject. As for the latter, the reader may refer to *H1* and to my book *Physique du Sens* [5] in which he/she

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[1]Petitot [1988], "Approche morphodynamique de la formule canonique du mythe", *L'Homme*, 106-107, 24-50.

[2]Petitot [1995], "Note complémentaire sur l'approche morphodynamique de la formule canonique du mythe", *L'Homme*, 135, 17-23.

[3]E. Köngäs and P. Maranda [1971), *Structural Models in Folklore and Transformational Essays*, La Haye, Mouton.

[4]L. Scubla [1996], *Histoire de la formule canonique du mythe et de ses modélisations*, Dissertation, Paris, École des Hautes Études en Sciences Sociales.

[5]Petitot [1992], *Physique du Sens*, Paris, Éditions du CNRS.

will find the essentials of the mathematical, physical and epistemological bases of Thomian morphodynamics.

In order to ensure that this paper is somewhat self-contained, I will, however, present first a few brief reminders.

1. EPISTEMOLOGICAL FOREWORD

1.1. The problem of the experimental method

Many adversaries of the *CF* deny it any value whatsoever and even denounce it as absurd. They maintain that its applications project and superimpose a preconceived formal structure on mythical reality. This criticism is quite serious and justified to a large extent. It raises a fundamental problem: that of the experimental method in structural analysis.

It is, indeed, impossible to come up with a direct experimental confirmation of formal structures of meaning, such as the *CF* : this is an aspect of the hermeneutic circle. Due to the "resonant" character of meaning, such structures can always be recognized projectively in data. Applying a form of meaning to an empirical content *always makes sense* and therefore has an interpretative, not an objective, status. This justifies the objections regarding superimposition. But does it mean that structuralist approaches of meaning have to be abandoned ? I do not think so. It is in fact possible to break the hermeneutical circle by conferring on structures a mathematical status rich enough to enable them to *generate models* that can be compared to empirical data in accordance with suitable experimental methods. In order to do so, we have to first probe considerably deeper into the conception of structures and their models. For as long as the *CF* is considered a simple *elementary* algebraico-combinatorial structure, there is no way that it can generate models, and consequently we come up against anti-structuralist criticism (see *H2*).

1.2. The problem of modeling

The crucial problem of modeling generally tends to be very poorly understood in the Humanities. It is rather naïve to believe that a mathematical model can be derived inductively from the empirical observations and theoretical conceptualizations of phenomena. History of formalized sciences (above all physics) demonstrates that such is not the case. Briefly summarized, what happens is rather the following. The scientific description of the empirical data leads, through abstraction and categorization to basic concepts *specific* to a discipline. These are "regional" or "domain dependent" concepts, even if they are related to rational principles of universal significance (based on categories of formal ontology such as object, property, relation, whole/part, continuous/discontinuous, invariance, cause, etc.). At this level, each discipline uses a specific operational language imposed by field.

The first stage of mathematization then comes into play. It concerns the *mathematical schematization* of the regional concepts and consists of a *mathematical interpretation* of their theoretical content. Such an interpretation cannot be inductive; it is rather abductive (in Peirce's sense), and consequently, not experimentally falsifiable. Of course, it makes use of available tools. Riemannian geometry and tensorial calculus, upon which general relativity is based, both existed long before Einstein and were elaborated by pure geometers (from Riemann to Levi-Civita). Physicists, however, never spoke of superimposing mathematical structures on data, nor did they propose a purely physics-specific mathematics. Likewise, the geometrical concept of connection that underlies contemporary gauge theories (from Yang-Mills' theories to more recent works by Witten) was conceived of by Elie Cartan at the beginning of the century. Again, no physicist ever put mathematicians on trial for being too avant-garde.

From the mathematical schematization of the theoretical concepts specific to a discipline, many different accurate models can be elaborated and measured against experience. It is impossible to empirically falsify (in Popper's sense) a purely conceptual description (in fact, that is what enables Humanities to theorize without risk). Only explicit and effective models can be falsified. But this is only possible *if theoretical contents are transformed into generative algorithms*. Every concept is an unknown algorithm and the invaluable role of mathematics has always been to substitute algorithms reconstructing phenomena for purely conceptual contents.

Thus we see that in properly formalized sciences, the *conceptual subsomption* relationship of empirical diversity under a theoretical unity admits a *converse* relationship unfolding a *constructed* diversity that can be measured against the *given* empirical diversity. The way in which this correspondence — that is, modeling — is related to the general theoretical concepts and principles, guarantees its explanatory power. Modeling aims to reconstruct the phenomenal diversity of a domain of reality from its constituent concepts. It solves *the inverse problem* to that of conceptual subsomption. *In science, conceptual analysis must be convertible into computational synthesis*. Obviously, this is only possible because phenomena are only phenomena and not ontological "an sich" realities.

The purpose of morphodynamical models is to achieve such a program for semio-narrative structures [6].

1.3. The *CF* as a "structural equation"

In *H2* I reconsidered the central issue of the *formal* status of the *CF* as a constituent element of what Claude Lévi-Strauss has referred to as the "grammatical discipline" of myths. The problem is : how can we do justice to the richness of the *CF at the level of pure form*?

I showed that to interpret the *CF* as an elementary structure (Klein group, semiotic square, etc.) is insufficient for at least three reasons.

(i) First of all, the *CF* expresses a *coupling* between two qualitative oppositions, which is much more than a simple cartesian product (Klein group).

(ii) Secondly, the difference between what Lévi-Strauss called the "term value" and the "function value" of the *CF*'s constituents comes from that between syntax and semantics in structuralist models. It cannot be represented in an elementary structure.

(iii) Thirdly, the *CF* deals with sets of mythological variants. It is not "intra", but rather "inter"-mythical; not "local" but "global". And this property cannot be encoded in an elementary structure either.

In order to overcome these difficulties and make the formula *both* theoretically compatible with the categories and principles of structuralism (which eliminates the risk of superimposition), *and* algorithmically generative of models (which makes it possible to resort to the experimental method), I suggested to conceive of the *CF* as a sort of basic "equation" whose "solutions" consist of different formal semio-narrative structures.

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[6] See Petitot [1992].

The basic equations in physics (for instance, Newton's equation in rational mechanics, Navier-Stokes' equation in hydrodynamics or Feynman's path integrals in quantum field theory) express very general principles (principles of relativity, symmetry, least action, conservation, causality, etc.). Their solutions, however, are remarkably precise models of an amazing phenomenal variety. They encode in their very compact and universal mathematical form an unpredictable universe of empirical diversity and complexity. This astonishing ability — this "miracle" — earned them the title of "intelligent equations."

The *CF*, in my view, is an "intelligent formula." It, too, encodes in a compact algebraic form expressing general structuralist principles of conflict and stabilization, an unpredictable universe of empirical diversity and complexity. Rather than a formula, I consider it a "structural equation" of mythological syntax, an equation to which myriads of actual myths offer so many empirical "solutions." This explains the risk of superimposition. Indeed, by making the *CF* into an elementary structure, "*equation*" *and* "*solution*" *become one and the same thing* with the result that the fundamental dialectic between subsomption and modeling is destroyed. The elementary structure is then uniformly "superimposed" onto empirical diversity. Instead of modeling its complexity, it reduces it to a repetitive archetype.

1.4. Towards a morphodynamical approach

In order to maintain the difference between the universal form of the *CF* "equation" and the vast diversity of its "solutions," its constituent relations must be interpreted mathematically. To do so, an adequate mathematical universe must be chosen. To the extent that the *CF* intimately depends on the theoretical concepts and principles of structuralism, I considered relevant to adopt the mathematical universe of *dynamical structuralism* and to model structures using morphogenetic models of differentiation, organization and regulation. Other choices undoubtedly exist, but the morphodynamical option has the advantage of being based on powerful mathematics whereby the theoretical problems of structuralism can be adequately interpreted and structural categories properly schematized (see *Morphogenèse du Sens* and *Physique du Sens* [7]).

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[7]Petitot [1985], *Morphogenèse du Sens*, Paris, Presses Universitaires de France and [1992].

The main hypothesis, as we have seen, is that the *CF* is more than the expression of a simple semantic analogy between two qualitative oppositions, namely a coupling between two oppositions defined on different semantic dimensions. In the morphodynamical schematization, the binary paradigm constituted by a simple qualitative opposition is modeled by a singularity called "cusp." The analogy between two oppositions is therefore modeled by a simple isomorphism between two cusps. But a coupling between two qualitative oppositions is modeled by a singularity called "double cusp" in which both cusps *interact*. However, the double cusp is definitely more complex than the simple cartesian product of two cusps. Out of its complexity a considerable number of different semio-narrative structures can be drawn, all of which are solutions to the problem of coupling ("modes" of coupling).

It is this idea that I will attempt to develop below.

2. REMINDERS CONCERNING MORPHODYNAMICAL MODELS

2.1. Semio-narrative models

The models we use are semio-narrative ones. They are based on the hypothesis that there are:

(i) *terms*, at the *syntagmatic* level, that is, *actants* in the sense of an actantial syntax (to be distinguished from actors or characters that usually syncretize several actants and tsupport thematic roles),

(ii) semantic *functions*, at the *paradigmatic* level, that depend on codes (in Lévi-Strauss' sense) belonging to deep structures: they are values categorizing the continuous substratum of paradigms into discrete units.

That an actant t takes on a semantic value v is written $F_v(t)$ in the *CF*.

The problem, which is an extremely difficult one, lies in holding together the paradigmatic (semantic "codes") and the syntagmatic (actantial interactions) levels. A basic thesis of semio-narrative structuralism is that paradigmatic semantic relations can only be implemented through actantial syntagmatics. Semantic values are "confined", "invested" in the actants and circulate through their interactions.

Three theoreticians have played a crucial role in elucidating these relationships: V. Propp, C. Lévi-Strauss and A.J. Greimas[8]. With his grammar of functions, Propp overly dissociated the narratively dominant actantial syntax from the semantic content. All too often he reduced the latter to simple thematic roles. On the other hand, by focusing dually on the paradigmatic axis and its projection on the syntagmatic axis, Claude Lévi-Strauss somewhat under-estimated the problem of actantial syntax. The synthesis was achieved by Greimassian theory which showed in detail how actantial syntax could handle logico-combinatorial operations on paradigmatic values.

We readily acknowledge that, with regard to Claude Lévi-Strauss's *Mythologiques*, the semio-narrative point of view may be somewhat biased in that there appear to be very few constraints applicable to the actants' identity in myths. Metamorphoses systematically occur in myths, which are very different from those found in fairy tales and, a fortiori, in more realistic stories. But these different types of identity concern the actors (characters) rather than the actants themselves.

2.2. Morphodynamical models

Morphodynamical models rest on three basic hypotheses that make explicit the constituent operations of the components $F_v(t)$ in the *CF* [9].

1. First of all, we hypothesize that the semantic axes (the continuous substrata that the values discretize) constitute spaces, referred to as *internal spaces*. Each dimension of such a space *M* usually will be the continuous substratum of a qualitative opposition (*x*/*y*) (of a distinctive feature +/–), somewhat like in elementary geometry an axis of coordinates is divided into a positive half and a negative half. When there are several dimensions, several semantic axes are likely to be coupled. Coupling entails that the decomposition of *M* is not a simple combinatorics of + and – (like two Cartesian axes split a plane into four quadrants ++, +–, –+, —). But the space *M* is still subdivided into domains (categories) whose centers represent values and the boundaries, relationships (usually of opposition) between

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[8]These works are master pieces of structural epistemology and methodology beginning with the works of Saussure, the Prague Circle, of Husserl's third *Logical Investigation*, Jakobson, Hjelmslev and Brøndal.

[9]This section summarizes *H2*.

values. It is, as we say, *stratified*, its stratification being more complicated than that brought about by hyperplanes of coordinates.

2. The syntactic confinement of values, their embodiment in an actant, is then expressed by a *variational principle*, referred to as the model's *internal dynamics*. That is the second basic hypothesis. We assume that there exists a potential function $f(x)$ defined on the internal space M, and that one of its minima represents the actant a under consideration. If this minimum corresponds to a point on M belonging to the domain of the value x, it will be said that x "invests" a or that a "represents" or "actantializes" x. If f has several minima, there are therefore several actants a, b, c... invested with different values x, y, z..., and the relationships between the minima become actantial relationships "actantializing" relationships between values. This schematizes the paradigmatic organization of the semantic substrata, and the dialectic between the paradigmatic-semantic and the syntagmatic-actantial dimensions of semio-narrative structures.

3. In order to be able to move from static relations to dynamic interactions, the potential f must, moreover, be able to evolve over time. This means that the potential f must be temporally parametrized: $f(x)$ must be of the form $f_t(x) = f(x,t)$. Furthermore, several parametrizations, which correspond to as many different scenarios of interaction, must be possible. This brings us to the third basic hypothesis: there is a second space (W), referred to *external space*, which parametrizes the internal dynamics : f has the form $f_W(x)$. The temporal evolutions of f are therefore *temporal paths* in this external space : f has the form $f_{W(t)}(x)$.

Such models where the categorization of a continuous substratum space into sub-domains (values defined by reciprocal determination) is generated by a family of generating potentials, have become widespread in contemporary cognitive sciences. If the categorization process is implemented in a network of formal neurons, the generating potentials become true potentials in the physical sense of the term, i.e., "energy" functions whose minima [10] determine the terms of the categorization. For example, in the case of a neural net R implementing a phonetic categorization, M will be the space of the instantaneous states of R, and W a space of acoustic cues (voicing, articulation point, etc.) controlling the phonetic percepts. The minima A_w, B_w, etc. of $f_w(x)$ define the phonemic values as well as the relations of dominance between them. For each w, a phonemic value

[10]In physics, to minimize energy is the basic variational principle.

will be dominant, and therefore, actualized, the others remaining virtual. But for different w, a transition of dominant values will occur. Each value A_w will therefore have its own *domain of domination* W_A within W. These domains geometrize the reciprocal determination relationships between values. They constitute a partition of the external space W, that is an *external* categorization generated by the family of generating potentials $f_w(x)$. Their boundaries form a critical set K which stratifies W and materializes the categorization (any categorization can thus be identified with a set of boundaries in a substratum space). Therefore, M undergoes an internal categorization through the attraction basins of $f_w(x)$ and an external categorization through the catastrophic set K. This results in a subtle internal/external dialectic allowing for an externalization of the internal paradigmatic relations between values. The stratification (W, K) unfolded by the generating family $f_w(x)$ externalizes the internal paradigm P.

As we have seen, a syntagmation of the paradigm P involves introducing paths in the external space W. The crossings of K are therefore comparable to syntactic events that commute the values of P while at the same time sequentializing them. That is how the projection of the paradigmatic axis on the syntagmatic axis takes place in these dynamical models.

2.3. The cusp example

The best-known example of such a model is the *cusp* catastrophe unfolding a cusp singularity. The normal form of the family of potentials $f_w(x)$ is given by the formula :

$$f_w(x) = x^4 - ux^2 + vx$$

The internal space M is one-dimensional (x coordinate). It serves as a substratum to a qualitative opposition. The external space W is 2-dimensional (u, v coordinates). Depending on whether $u > 0$ or $u < 0$, $f_w(x)$ has either a single minimum or two conflicting minima separated by a maximum. For this reason the u parameter is referred to as the "splitting factor". It controls the formation or neutralization of a conflict between two determinations. As for the v parameter, in the case where $u < 0$, it controls the domination of one of the minima over the other. It is called the "bias factor". The cusp sheets correspond to the extrema of $f_w(x)$. For $u < 0$ there are two attracting sheets (minima) separated by a repelling intermediary sheet (maximum). The three sheets connect along a fold curve and fuse for $u = 0$, and for $u > 0$ just a single undifferentiated sheet remains.

The (W, K) stratification is, therefore, essentially composed of a 2-dimensional space W made up of a neutral zone and a conflict zone. The latter is limited by bifurcating strata K_b along which one of the minima disappears by collapsing with the maximum and becomes captured by the other minimum. It is separated into two domains by an interface K_c (a threshold corresponding to a conflicting equilibrium between the two minima). The bifurcating strata K_b are the projections of the folds of the cusp surface (see figure 1).

Fig. 1

Figures 2 and 3 illustrate an example of a path in the external space W. This corresponds to a capture catastrophe.

Fig. 2

Fig. 3

As a model of structural relations, the cusp schematizes a qualitative opposition A/B. On one side of K the A value dominates. On the other side, it is the B value that dominates. A path that crosses K therefore corresponds to an $A \rightarrow B$ type of syntagmation: the initial state where A dominated over B is replaced by one in which B dominates over A.

In *Physique du Sens,* I explain the way in which the cusp models the syntactic handling of qualitative oppositions both through Subject-Antisubject ($S/\overline{S}$) actantial conflicts and through Subject-Object (S/O) conjunctions.

3. PIERRE MARANDA'S MODEL OF MEDIATION

3.1. Mediation

In his Dissertation, Lucien Scubla presented detailed comments on Pierre Maranda and Elli Köngäs's model, which describes the *CF* as an algorithm *mediating* an initial contradiction. The *CF* is thus

> "entirely deduced from the single idea that mythical operations aim to resolve a contradiction through a process of mediation and that the formula describes the passage from the initial state to the outcome of such a mythical transformation".[11]

I think that a reconsideration of the mediation model allows us to improve significantly on the double cusp model presented in *H1*.

Let us begin with the standard form of the *CF* :

$$F_x(a) : F_y(b) :: F_x(b) : F_{a^{-1}}(y)$$

1. The first part of the formula $F_x(a) : F_y(b)$ corresponds to a qualitative opposition both between the terms *a*/*b* and the functions *x*/*y*. This means that two opposed *x*/*y* values (for example, Nature/Culture) are actantialized by a conflict between two actors *a*/*b* (for example, Dragon/Hero).

2. The second step is a *mediation* effected by *b*. Having initially embodied the "right" value *y*, the term *b* actantializes also the opposite "evil" value *x*. This explains the third component in the formula, $F_x(b)$ corresponding to the mediation.

3. Obviously, this mediation does not imply a simple swap, a switch-over between *a* and *b*, *a* in turn mediating the opposition by actantializing symmetrically the "right" value *y*. The appropriation of the initially negative value *x* by the positive term *b* is accompanied by an elimination of the term *a* and a sort of "assumption" (an "apotheosis" in Scubla's words) of the positive *y* value. *a* is eliminated as a term value *and it is this operation itself which becomes now a value* a^{-1}. At the same time, the assumption of *y* is an "embodiement" of it, which explains the inversion of the term value and the function value characteristic of the terminal twist. This generates the fourth component of the formula : $F_{a^{-1}}(y)$.

Hence the presentation of the formula as a structure that is both static and dynamic:

$$A : B :: M : X$$

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[11] Scubla [1996], p.210.

$$A/B \Rightarrow M \Rightarrow X$$

where A/B (i.e. $A : B$) is the initial opposition $F_x(a) : F_y(b)$ (i.e. x/y is represented by a/b), M is a process of mediation and indifferentiation, and X is an inversion of the mediation and a new process of differentiation generating a new qualitative opposition between A'/B' (looping twist).

We will now develop the idea that a morphodynamical formalization of Maranda's model automatically leads to a structure embedded in the double cusp catastrophe.

3.2. The main ideas of the model

We will develop the following main ideas.

1. With regard to the *CF*, the cusp models the *dynamics* of the conflict $F_x(a)/F_y(b)$. It formalizes:

(i) the *genesis* of the conflict between a/b,

(ii) the possible *neutralization* of the conflict,

(iii) the possibility for an actant (for example, b representing y) to *wind around* the organizing center of the cusp and to represent the opposite function x (this corresponds to a transformation $F_y(b) \rightarrow F_x(b)$). This mediation can, obviously, be symmetrized (transformation $F_x(a) \rightarrow F_y(a)$). In conformity with Maranda's idea, a mediation is therefore not a logical operation but rather the *exploration of a conflict dynamics*.

2. The inversion of the term and function values as well as the "looping twist" characteristic of the *CF* are to be interpreted as an "*internalization*" of the external parameter (that drives the mediation) into a new internal variable (in the sense of the duality between external and internal spaces). This automatically leads to the double cusp model.

IV. ADDRESSERS AND EXTERNAL DYNAMICS

Let us begin by clarifying certain constituent operations necessary to the proper functioning of Maranda's model.

1. First, a specific status must be attributed to the concept of "representation" or "actantialization" of a value *v* by an actantial term *t (i.e.* a $F_v(t)$ component). We have seen that in morphodynamical models, actantial relations are described by the relationships between minima of generating potentials. The relationship between an actant and a value is more Proppian-Greimassian than Lévi-Straussian as it is built through *value-objects*. Values are invested in objects, and it is the syntactic relationships of junction/conjunction between the subjects and anti-subjects and such objects that make them represent values. But what drives the syntactic relationships actantializing the relation to values? Simple "confinement" as described above is not sufficient.

2. The answer to this question lies in the fundamental actantial concept of *Addresser*.[12] Actantially speaking, the Addresser is the actant that, on the one hand, provides and guarantees values and, on the other hand, *modalizes* the subjects (e.g. casts them in the "will" mode), i.e., and *controls their actantial trajectories*. For example, in myths of the Hero/Princess/Dragon type, the Addresser is the royal figure of the father. But he has not to be necessarily anthropomorphic. The purest narrative example of an Addresser I ever met is the eagle's flight in Stendhal's *La Chartreuse de Parme* that persuades Fabrice, in a flashing and imperious way, to leave Milano for Waterloo.

When the Addresser is external to the Subject actant, he modalizes him deontically and casts him in the "duty" mode (the Subject acts according to the program assigned to him by the Addresser). When the Addresser is internal to the Subject the modality becomes that of "will" (the Subject controls intentionnally his actantial trajectory, his "will" acting as a self-destination).

In my Dissertation on actantial morphodynamics,[13] I proposed that the connection between value, value-object, Subject and Addresser be modeled as follows:

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[12]The French name "Destinateur" would be more convenient for it includes the connotation of destiny and fate.

[13]Petitot [1982], *Pour un Schématisme de la Structure*, Dissertation, Paris, École des Hautes Études en Sciences Sociales.

(i) The value v is identified with the *threshold* separating the subject S from the object O and the capture of O by S is therefore ipso facto a capture of v by S.
(ii) Moreover, the value is also the semantic content of the internal axis on which the junction (disjunction/conjunction) S-O is defined.
(iii) The Addresser controls the external trajectory that leads from the disjunction S/O to the conjunction S-O.

Per Aage Brandt later showed in his reference work *La Charpente Modale du Sens* [14] that generally speaking, Addressers can be described morphodynamically as *control* dynamics in the *external* spaces of models. External dynamics drive the actantial interactions (actants' "programs" of action).

When a Subject embodies a value, an Addresser assigns him the mission of outdoing antagonistic forces represented by an Anti-Addresser. Subject / Anti-Subject conflicts in the internal spaces are therefore controlled by conflicts Addresser / Anti-Addresser *dynamics* in the external spaces (see figure4).

Figure 4

To say that a Hero t "embodies" a value v, means that he becomes his own Addresser and controls his external dynamic causing v to triumph.

3. There exists an essential connection between Addresssers and the *axiological* dimension of any conflict. To say that a value is axiologically positive, means that it drives the modal intentionality of the external dynamic. In other words, axiology *polarizes* external spaces positively / negatively and, through dynamical attraction / repulsion interplays, generates external dynamics. For example, in the cusp model, the half axis $v > 0$ drives the Addresser's dynamics ($\overline{S}$ is captured by S) while the half axis $v < 0$ drives the Anti-Addresser's one (S is captured by $\overline{S}$).

5. THE PARADOXES OF MEDIATION

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[14] Brandt [1986], *La Charpente Modale du Sens*, Dissertation, Université de Paris III. Published in 1992 by John Benjamins.

Now let us apply the simplest model, namely the cusp, to the problem of mediation. As we will see, it solves half of the problem, i.e. the first twist in the *CF*, but leads to paradoxes that can only be solved by complexifying the model.

5.1. A model of mediation as twist

The cusp model makes it very easy to interpret the first twist, that is, the third component $F_x(b)$ of the *CF* and the "Moebius" effect $F_y(b) \rightarrow F_x(b)$. The characteristic of the cusp is the fact that an external path can *wind around the organizing center* and, like a loop, come back to the starting point (see figure 5).

Fig. 5

Such a cycle is very particular. It is a "marked" cycle generating what Thom named an "actant confusion" (see figure 6).

Fig. 6

The "marked" cycle perfectly models the mediating component $F_x(b)$. Having bypassed the organizing center, the trajectory of *b* (which captured *a*) leads it from sheet *y* to sheet *x*, which corresponds exactly to $F_x(b)$. Moreover, mediation takes place by *neutralizing* the threshold separating *x* and *y* : the single sheet beyond the organizing center is neutral in relation to the *x*/*y* opposition.

2. Mimesis and mediation

But in this cusp model, mediation proves paradoxical for at least two reasons. First, as we have seen, *a*'s capture by *b* transforms *a* into a value-object for *b*. But, whereas *b* is a "subject" actantializing the value *y*, *a* is more of an "anti-subject" (enemy, adversary) than a value-object to be conquered.

Furthermore, the actantial confusion involved in mediation implies that *b* has, in Thom's terms, "alienated itself" in the role of *a*. In other words, *by virtue of mediation* *itself*, the $S/\overline{S}$ opposition typifying the *b/a* opposition, becomes somewhat analogous to a *mimetic* relationship in Girard's sense (see figure 7).

Fig. 7

This seems to be a deep and well-grounded anthropological phenomenon, essential to understand rituals. But it is nevertheless rather paradoxical.

6. BEYOND THE MEDIATING CONTRADICTION

As we can see, attempting to solve the contradiction $F_x(a)/F_y(b)$ by means of a mediation $F_x(b)$ leads to an even more serious contradiction, logical this time, concerning identity. This new contradiction needs also to be solved.

6.1. Introducing value-objects

The first possibility is to introduce a value-object which the subjects $\overline{S}$ *(i.e. a)* and S *(i.e. b)* are competing to conquer. This is the case in typical Hero/Princess/Dragon myths.

In *Physique du Sens* I showed that such actantial scenarios are accurately described by the Thomian transfer catastrophe called "butterfly" which expresses the way in which a third actantial place (that of the value-object) O_v is transferred from an $\overline{S}/O_v$ cusp to an S/O_v cusp. If the external dynamics of the Addressers that control the subjects' intentional duty and will are introduced, the domination of S over $\overline{S}$ is expressed by the capture S-O_v (which is a tearing away, a disjunction, $\overline{S}/O_v$).

In much the same way as the cusp $x^4 - ux^2 + vx$ is the universal unfolding of the x^4 singularity, the butterfly catastrophe is the universal unfolding of the x^6 singularity. Its external space is 4-dimensional (coordinates t, u, v, w). It can be reduced to three dimensions since it is only for $t<0$ that 3 minima occur. For $t>0$, the butterfly is simply a cusp. Figure 8 shows the evolution of the plane sections (v, w) of the catastrophic set K for $t = -1$, when u varies from 0.3 to –0.3. An initial cusp "emits" along one of its branches a "swallowtail" one cusp of which generates a new minimum, while the other cusp, called a dual cusp, generates a new maximum. The new cusp becomes progressively dominant while the initial cusp regresses. Finally, the latter collapses together with the dual cusp and we end up with a single cusp.

Fig. 8

Figures 9, 10 and 11 respectively show the evolution of the butterfly sheets above the (v, w) plane when u varies, the evolution of the internal potential along a transfer path (the intermediary minimum transiting from one minimum to the other is clearly visible) and the three phases in the transfer: emission-transit-reception.

Fig. 9

Fig.10

Fig. 11

One of the main interests of this model is that it obstructs the mimetic relationship $S/\overline{S}$. In fact, it contains a value which is not actantialized by O_v (as it can be in a Subject actant S or $\overline{S}$), but is simply "invested" in O_v. The object O_v does not itself represent the value, it only "confines" it. It is true that the relationships S/O_v and $\overline{S}/O_v$ are organized by cusp singularities, and that these cusps allow for confusion between actants but, insofar as the object O_v does not actantialize the value v, this "fusion" essentially expresses the fact that the Subject acquires the value v by captating the object O_v. On the other hand, the relation $S/\overline{S}$ is a *pure conflict*, without any organizing cusp. Actant confusion $S/\overline{S}$ (mimesis) is thus avoided. Figure 12 shows a very simplified section of the external space of the transfer catastrophe containing not only the bifurcation strata but also the conflict strata. The stratum of pure conflict $S/\overline{S}$ is clearly visible.

Fig. 12

Another point of interest in this model is that S represents the value v by capturing an object O_v that "confines" v. In other words, through the mediation of O_v, a component of the CF such as $F_y(b)$ is now intepreted *as a trajectory in the external space* (i.e. as a narrative program of conjunction) $S = b–O_y$). This leads to another means, more apt in my view, of resolving the mediation contradiction.

2. Identity reintegration and value incarnation

The origin of the mediation contradiction lies in the fact that actant confusion causes actant *b* to occupy the *x* value's sheet initially occupied by *a*. But *b* occupies this position as a result of a *dynamical process* which is fundamentally different than that which determined *a*. In the cusp model, there are essentially three dynamical scenarios leading to an actant's occupation of a sheet (i.e. to a relationship of the $F_v(t)$ type):

(i) The actant occupies statically its natural sheet: this corresponds for instance to the $F_x(a)$ or $F_y(b)$ components of the *CF* (see figure 13a).

(ii) The actant occupies its natural sheet, but after having captured the anti-actant (see figure 13b).

(iii) The actant occupies the sheet opposite to its natural one following a process of actant confusion ($F_x(b)$ in the *CF*) (see figure 13c).

Fig. 13

A first means of resolving the paradox of mediation would therefore be for actant *b* to reintegrate its natural sheet after the stage (iii) through a sort of "tunnel effect" (see figure 14).

Fig. 14

Actant *b* thus reintegrates its identity having *doubly* eliminated *a*. In fact, not only it captured *a*, but it transcended the mimetic effect connected with this capture. It threw off the residue of *a* in which it was alienated and henceforth integrated the "evil" value of *a* to the "good" one *y*. This expulsion of *a* can be identified with the term a^{-1}, and can be referred to as an "assumption" of *b* and *y*. Actually, insofar as the provider and guarantor of values is the Addresser, *b shifted from the status of Subject to that of Addresser*.

The fourth component of the formula $F_{a^{-1}}(y)$ could therefore be interpreted as a "reprogramming" of identity which enables to disalienate the alienation constituent of mediation. Thus the idea that actant confusion leads to a *new* differentiation, which is no longer a simple *x*/*y* static opposition but a complex *dynamical process*.

If this idea of a "tunnel effect" seems too sophisticated, consider that *b* simply continues its cycle and, after a sufficient delay, reintegrates its natural sheet by a standard catastrophic jump (see figure 15).

Fig. 15

This would give us an even more simple interpretation of the fourth component of the *CF*. Indeed, since the catastrophic jump takes place between the mediation $F_x(b)$ and a return to the initial component $F_y(b)$, it could be said that actant *b* appropriates *the entire x/y opposition* in order to "incarnate" it. Actant *a* is therefore completely eliminated, which is expressed by the term a^{-1}.

This gives us the global dynamical schema in figure 16.

Fig. 16

6.3. Summary of the cusp model

The stages in the *CF*'s cusp model can therefore be summarized as follows:

1. $F_x(a)$ means that *a* occupies a natural sheet that makes it representative of the *x* value.
2. In the same way, $F_y(b)$ means that *b* occupies a natural sheet which makes it representative of the *y* value.
3. $F_x(a) : F_y(b)$ means that these two representations are in conflict : it corresponds to the double-sheeted conflict zone inside the cusp.
4. The mediation $F_x(b)$ means two things:
   (i) that *b* actantially dominates *a*;
   (ii) that it alienates itself in the value *x* represented by *a*.
5. The twist component $F_{a^{-1}}(y)$ means also two things :
   (i) That *b* reintegrates its identity through the "catastrophic" expulsion of *a* (term a^{-1});
   (ii) The assumption of *b* as an Addresser "incarnating" the *y* value.

4. The problem of internalization

The dynamical scenario leading from input $F_x(a)$ to the output $F_{a^{-1}}(y)$ is rather subtle (mimetic mediation + identity reintegration). The external dynamics controlling *b*'s actantial modalities and revolutions displays the Addresser's action. Hence, the issue of whether or not *this dynamics can itself be represented statically on the condition that the internal spaces defining the relationships are complexified.*

The most natural idea consists in *internalizing the external dynamics*. It leads straightforward to the double cusp model.

7. INTERNALIZING EXTERNAL SPACES

7.1. The problem

Let us suppose that in the cusp example we wanted a more complicated internal dynamic to necessarily lead from an initial situation $F_x(a) : F_y(b)$ to an outcome where *b* has captured *y*. A simple way to do this is to consider a section of the cusp $X^4 + uX^2 + vX$ with $u < 0$ (for example, $u = -1$), with v varying from negative to positive values, and to treat v as a new *internal* variable.[15] More precisely, we want a value close to a v_0 value of v located on the right outside the cusp to attract the trajectory γ (see figure 17).

Fig. 17

When we try to put it into practice, this natural idea proves rather tricky technically because it is difficult to intuitively master the couplings of variables that come forth. We can, however, provide some fairly simple indications with the help of illustrations.

7.2. The energy landscape $f(X,v)$

Let us begin by making v an internal variable in the cusp model by treating the potential $f(X,v) = X^4 - X^2 + vX$ no longer as a family of potentials $f_v(X)$ with an internal

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[15] The internal variable X is the support for the opposition between the x/y values that label the cusp sheets.

variable X dependent on an external parameter v, but rather as a unique potential with *two* internal variables.

If we look at the graph of $f(X,v)$, we can see that it is composed of:
(i) a main "valley" (the principal minimum m_p of $f_v(X)$) which slopes downward, and
(ii) a secondary valley (the secondary minimum m_s of $f_v(X)$) which slopes upward and disappears through bifurcation by coalescing with the maximum (see figure 18).

Fig. 18

The bifurcation takes place for the value v_{crit} of $v>0$ for which $f_v'(X)$ has a double root (inflection point where m_s collapses with the maximum M). v_{crit} is the solution to the system of equations :

$$\begin{cases} f_v'(X) = 4X^3 - 2X + v = 0 \\ f_v''(X) = 12X^2 - 2 = 0 \end{cases}$$

Its numerical value is $v_{crit} = 0.54$.

The valleys are generated by the minima of $f_v(X)$, i.e. by the solutions to

$$f_v'(X) = 4X^3 - 2X + v = 0$$

Figure 19 represents them in the plane $(y=f(x),v)$ with their separating ridge. It clearly shows the descent of m_p, the rise of m_s and its disappearance by collapse with the maximum M (bifurcation).

Fig. 19

7.3. Making capture necessary

If we want the capture scenario driven by the *external* dynamics of the initial internal variable X to be now driven by the *internal* dynamics of the new internal variable v, the energy landscape must be modified in two ways:

(i) the main valley also must admit a minimum (in the neighbourhood of v_0), but relative to the v dimension (and no longer to the X one);

(ii) the slope of the secondary valley must switch downward.

To make the secondary valley slope downward, we can add a term $-av$ (independent of X) to $f_v(X)$, where a is a coefficient accentuating the slope of the valley m_s). For $a = 1$, we get figures 20 and 21, which show that the valley m_s has indeed become downward-sloping.

Fig. 20

Fig. 21

But in order for the main valley m_p to have a minimum near v_0, we need a little more. The simplest is to introduce a quadratic term in $(v-v_0)^2$ with v_0 far enough removed from v_{crit}. This gives us the potential :

$$f(X,v) = X^4 - X^2 + vX + (v - v_0)^2$$

For $v_0 = 1.5$ (remember that $v_{crit} \approx 0.5$), we get figures 22 and 23.

Fig. 22

Fig. 23

This simple quadratic transformation of the standard cusp model ensures that the m_s valley slopes downward. This comes from the fact that we can add a constant term $c(v)$ to the potentials $f_v(X)$ without altering their gradient dynamics. To make the m_s valley slope downward, the constant $c(v)$ simply needs to decrease sufficiently until m_s bifurcates (for $v_{crit} \approx 0.5$). This was the case for $c(v) = -v$. But it is also the case for $c(v) = (v-v_0)^2$ when v_0 is sufficiently greater than v_{crit}.

The minimum (X_{min}, v_{min}) of $f(X,v)$ is then represented by the system of equations:

$$\begin{cases} \dfrac{\partial f}{\partial X} = 4X^3 - 2X + v = 0 \\ \dfrac{\partial f}{\partial v} = X + 2(v - v_0) = 0 \end{cases}$$

In the case $v_0 = 1.5$, we get $v_{min} \approx 1.81$.

7.4. Coupling internal variables

The potential $f(X,v) = X^4 - X^2 + vX + (v-v_0)^2$ can be interpreted as a *coupling* between two independent potentials $f_1(X) = X^4 - X^2$ and $f_2(v) = (v-v_0)^2$ respectively defined along the internal axes X and v. f_1 rules a pure conflict (the x/y conflict on the X axis), and f_2 a state of quadratic equilibrium that attracts v toward v_0. The "free" potentials f_1 and f_2 are linked in f by a coupling (or "interaction") term $f_{int} = vX$. This multiplicative coupling is the simplest possible. It is, in a sense, "minimal."

The minimal multiplicative coupling implies that v actually operates as a bias factor for f_1 [16] and that therefore, when it is attracted by v_0, v drives the a/b conflict towards a capture catastrophe, i.e. towards the domination of b over a on behalf of the y value. The idea of internalization is therefore to introduce a dynamics $f_2(v)$ for v and to couple it with the dynamics $f_1(X)$ in order to force the latter to evolve.

7.5. Slow and fast dynamics

In this type of internalization of the external parameter v we do not take into account the fact that, in the standard model $f_v(X)$, the internal dynamics on X is a "fast" internal dynamics that "instantaneously" projects the representative point X onto a minimum of $f_v(X)$, i.e. onto the surface of states, while the external dynamics on v is a "slow" dynamics that makes these minima evolve on the surface. The system $f(X,v)$ is what is called a *slow/fast system*, also referred to as "adiabatic", that is, a system with two very different time scales. If we wish to take this slow/fast difference into account in the internalization, we simply need to change the v scale (see figure 24).

Fig. 24

We will not take this problem into consideration here.

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[16] f_{int} also operates on f_2, but since f_2 is a quadratic potential, this does not produce any dynamical effect.

7.6. The meaning of internalization: inverting a term value and a function value

Let us clarify the structural meaning of the procedure of internalization. The $f(X,v)$ energy landscape now describes the capture of actant a by an actant b as a function of *two* internal parameters. The first, namely X, supports the opposition of values x/y. The second, namely v, on the contrary is the support for what used to be the external capture of a by b, i.e. the domination of the y value over the x value. As we have seen, these external dynamics described how the Addressers modalize the subjects.

Internalization therefore means:

(i) that the process of capture itself *becomes a value*, in other words, the "triumph" itself is a value — whatever the content of the x/y values may be;

(ii) that actant b was enriched by the "triumph" involuting (internalizing) the role of Addresser. b becomes its own Addresser during the end of the canonical process and its intentionality becomes involuted in the certainty of victory, instead of becoming externalized in a quest. In this sense, the y value is "incarnated" by b.

Such is the dynamical interpretation that we give of the inversion between term value and function value: a y value is transformed into a term (is actantialized) through the "triumph" of the term b which incarnates it; in other words, when the dynamical principle of its axiological domination is itself internalized into a value.

This provides new elements for interpreting the fourth component $F_{a^{-1}}(y)$ of the *CF*. It can be said that during the capture process, the elimination of a — that we will note $F_{a^{-1}}(y)^0$ — is also reinterpreted as a value (the anti-subject a becoming a scapegoat). This is a "precursor" of the final twist.

8. THE DOUBLE CUSP AND THE DOUBLE TWIST

8.1. Internalizing the conflict of external dynamics

Internalizing the bias factor v of the cusp model $X^4 - X^2 + vX$ thus accounts for a component of the $F_b(y)$ type expressing b 's "triumph", its capture of a, its "rising" to the rank of Addresser and the resulting inversion between *b* as term value and *y* as function value.

But there exists also a sort of "second degree" opposition between *a*'s and *b*'s intentional dynamics, i.e. between Addresser and Anti-Addresser. This means that in the internalization of the bias factor v, the $f_2(v)$ dynamics controlling v possesses *two* opposing minima v_{min} and $-v_{min}$. In other words, the dynamics $f_2(v)$ must *itself be a conflict dynamics*.

The simplest model is once again that of the cusp. As a "free" potential $f_2(v)$, we will therefore take the function $f_2(v) = v^4 - v_0^2 v^2$ whose minima are $v = \pm v_0$. For the sake of simplicity, we will use $v_0 = 1$ (which is sufficiently $> v_{crit}$). The minimal multiplicative coupling Xv therefore yields the potential :

$$f(X,v) = X^4 - X^2 + vX + v^4 - v^2$$

The analysis of this kind of potential, which expresses the simplest interaction between two pure conflicts, is already complicated. Its critical points are given in the system of equations :

$$\begin{cases} \dfrac{\partial f}{\partial X} = 4X^3 - 2X + v = 0 \\ \dfrac{\partial f}{\partial v} = 4v^3 - 2v + X = 0 \end{cases}$$

If we eliminate, we get an equation in X of degree 9 :

$$-4\left(4X^3 - 2X\right)^3 + 8X^3 - 3X = 0 .$$

But luckily, these 9 critical points are not all relevant. If we maintain the special coordinates (X,v), and if we follow the critical points (the two minima and the maximum) of $f_v(X)$, we get the swallowtail graph in figure 25.

Fig. 25

Figure 26 clearly shows the two antagonistic capture dynamics.

Fig. 26

8.2. The double cusp model

The above potential $f(X,v)$ is a partial unfolding of the singularity $X^4 + v^4$ which is none other than the *double cusp*, a singularity that expresses the coupling of two cusps, that is, the interaction between two oppositions with different supports.

This explains why in *Physique du Sens* and in *H1*, I proposed the double cusp as a global model (of which the *CF* is a sub-model) for the internalization of external dynamics. At the time, it may have seemed somewhat speculative. I am pleased that this new opportunity to reflect on the *CF* has enabled me to clarify this point.

The double cusp model couples two independent oppositions. In the sub-model currently under study, the first, X supported, is the initial axiological x/y opposition. The second, v supported, is the internalization of the opposition between the Addressers' dynamics (i.e. the actants' intentional dynamics). It expresses the conflict of the components $F_b(y)$ and $F_a(x)$ or, correlatively, of the components $F_{a^{-1}}(y)^0$ and $F_{b^{-1}}(x)^0$.

8.3. The loop in the first twist of the *CF*

How can actant confusion be taken into account in this internalized model? The simplest way is to internalize the bias factor v no longer as a linear dimension, but as a *cycle* $u^2 + v^2 = 1$ winding around the organizing center in the control plane (u,v). If we introduce an angular variable θ varying from 0 to 2π, we have $u = -\cos(\theta)$ and $v = \sin(\theta)$. For $\theta = 0$, the cycle takes off at $(-1,0)$. In order for this cycle to correspond to a valley in the energy landscape $f(X,v)$, a term that plays the role of $-v$ and $(v-v_0)^2$ in the preceding models must be introduced. The simplest is to take $-\theta$. This yields the potential :

$$f(X,\theta) = X^4 - \cos(\theta)X^2 + \sin(\theta)X - \theta$$

The analysis of its graph very clearly indicates how b's valley lands in position a at the end of a cycle. This leads us back to the third component of the *FC*, $F_x(b)$, i.e. the first twist (see figure 27).

Fig. 27

If we wish, parameters can be slightly adapted. In this model, $v = \sin(\theta)$ and therefore $|v| = |\sin(\theta)| \le 1$. If we want to stretch the cycle in the v direction, we can add a multiplicative factor and take $v = c \sin(\theta)$, or else use an oval-shaped cycle and not a circle-shaped one, etc. Qualitatively it does not change anything. Likewise, if the downwards slope imposed by θ seems a little too steep, θ can be replaced by $e\theta$.

In fact, the first twist gives all of its meaning to the precursor of the second twist $F_{a^{-1}}(y)^0$ (see section 7). Already the internalization of b's external dynamics could be interpreted as a $F_b(y)$ component whose effect on a was $F_{a^{-1}}(y)^0$. But this negation of a presupposes that the value x , initially actantialized by a, is in a sense "recovered." The component $F_{a^{-1}}(y)$ takes on its true meaning only *after* the actant confusion (first twist) $F_x(b)$.

8.4. The second twist

As we have seen in section 6.2, the second twist corresponds to b's reintegration of its identity, either through a sort of "tunnel effect", or through a catastrophe $F_x(b) \to F_y(b)$ closing the cycle, and expressing the fourth component $F_{a^{-1}}(y)$.

In order to stabilize the internal dynamics on this final state while guaranteeing the symmetry between b and a, a term $c\theta^4 - d\theta^2$ must be added (the constants c and d serve to decrease the slope, since $2\pi^4 \approx 195$).

This yields the potential :

$$f(X,\theta) = X^4 - \cos(\theta)X^2 + \sin(\theta)X + c\theta^4 - d\theta^2 \ .$$

If we want θ's minimum to be located for example at $2\pi+\pi/4$, we need $c \approx 0.0004e$ and $d \approx 0.04e$. In the following figures, we used $e = 5$.

The different stages of the process are clear. Figure 28 shows the evolution of the valley bottoms: *b* can be seen returning to its original sheet. Figures 29 and 30 show the loop taken by *b* from a variety of perspectives, as well as its different stages.

Fig. 28

Fig. 29

Fig. 30

This having been said, there is an aspect of the potential $f(X,\theta)$ that is not very satifactory. It mixes algebraic and trigonometric terms together. To remedy this incongruency, we can come back to the universal unfolding of the double cusp.

8.5. Double cusp and generalized couplings

In chapter VII of *Physique du Sens*, I explained at length the genesis of elementary catastrophes leading to the double cusp. I also summarized the work of C. Zeeman and J. Callahan on the highly complex geometry of the double cusp (which, in the list of singularities has symbol X_9). The double cusp $X^4 + Y^4$ is of a codimension 8 (in fact, reducible to 7). Its universal unfolding is given by the normal form :

$$f_w(X,Y) = X^4 + Y^4 + hX^2Y^2 + fX^2Y + gXY^2 + cX^2 + dXY + eY^2 + aX + bY$$

where w is the multiparameter (a,b,c,d,e,f,g,h) varying in the external space W.

If desired, a weight coefficient for Y can be introduced and Y^4 can be replaced by kY^4. Scaling Y in this way does not change anything qualitatively. A constant term p can also be introduced.

The universal unfolding f_w includes all the monomials in X and Y of degree ≤ 4, excepting those in the ideal (called the Jacobian ideal) generated by the partial derivatives $4X^3$ and $4Y^3$ of the organizing center $f_0 = X^4 + Y^4$, namely the monomials X^3Y, XY^3, X^3 and Y^3. This is due to the fact that in the functional space of functions, the universal unfolding of f_0 is a transverse section of the orbit $\overline{f_0}$ of f_0 under the group action defining its qualitative type. The Jacobian ideal of f_0 is the "tangent" space to $\overline{f_0}$ at f_0.[17] X^3 and Y^3 are therefore "tangent" to the orbit, and have to be excluded from the universal unfolding. But obviously, nothing prevents a general (not universal) unfolding of f_0 from having components tangent to the orbit.

Let us go back to the potential $f(X,\theta)$. For *small* θ we can expand the trigonometric functions cosθ and sinθ and use the approximations $\sin\theta \approx \theta$, $\cos\theta \approx 1-\theta^2/2$. This yields the potential :

$$f_w(X,\theta) = X^4 + c\theta^4 + \frac{1}{2}X^2\theta^2 - X^2 + X\theta - d\theta^2$$

i.e. an infinitesimal path in the universal unfolding of the double cusp with $k=c$, $h=1/2$, $c=-1$, $e=-d$, $d=1$, $f=0$, $g=0$, $a=0$, $b=0$.[18]

These potentials are semi-local : θ must remain small, whereas X can vary more freely. Furthermore, insofar as the potential is of degree 4 in θ, we could extend the approximation of sinθ to the order 3: $\sin\theta \approx \theta-\theta^3/6$ and that of cosθ to the order 4: $\cos\theta \approx 1-\theta^2/2+\theta^4/24$.

The problem is then to *globalize* such a description to the cycle that θ describes. This problem is rather tricky and can have several answers.

The easiest is simply to choose as internal space not the $\mathbf{R}^2$ plane (X,v), but rather a cylinder $\mathbf{R} \times S^1$ (S^1 = unit circle) of coordinates (X,θ). The potential $f(X,\theta)$ is then a potential on a part of the cyclinder.

By using the analysis made for the small values of θ, we can also consider for a given value θ_0 of θ an expansion around θ_0 and write $\theta = (\theta-\theta_0)+\theta_0$ with $\theta-\theta_0= s$ small.

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[17]For further precisions, cf. Petitot [1992].

[18]Although they sometimes share the same symbol, the general coefficients of the double cusp $f_w(X,Y)$ must not be confused with the constants appearing in the potential $f_w(X, \theta)$.

This yields the potential $f_{w(\theta)}(X,Y)$ of the double cusp which most closely approximates the potential $f(X,\theta)$ at θ_0.[19]

This is quite an interesting idea. The internal variable X that defines the underlying semantic axis of the *x*/*y* opposition and the variable θ which internalizes an external parameter are clearly distinguished. The double cusp $X^4 + Y^4$ is then considered a *generating* singularity, that is, a singularity whose universal unfolding is so rich that many different scenarios can be embedded in it. That was the main idea presented in *H1* : to treat the double cusp as a "*classifying space*" for the structures ruled by the *CF*. If we wish to internalize an external path of the cusp $(u(\theta),v(\theta))$ parametrized in a complex manner by θ (like a cycle ($u = \cos\theta$, $v = \sin\theta$)), we can consider for each value of $(u(\theta),v(\theta))$ the potential $f_w(X,Y)$ of the double cusp that best approximates it. This defines a sort of *infinitesimal internalization* of the path $(u(\theta), v(\theta))$, the global internalization being a sort of "integration."

This point of view is acceptable inasmuch as the condition of adiabaticity (see section 7.5) makes the external dynamics internalized by θ a slow dynamics relative to that on X.

If we apply this strategy to our example, we get the following results:

$$\begin{cases} \sin\theta = \sin\big((\theta-\theta_0)+\theta_0\big) = \sin(\theta-\theta_0)\cos\theta_0 + \cos(\theta-\theta_0)\sin\theta_0 \\ \cos\theta = \cos\big((\theta-\theta_0)+\theta_0\big) = \cos(\theta-\theta_0)\cos\theta_0 - \sin(\theta-\theta_0)\sin\theta_0 \end{cases}$$

If we expand $\sin(\theta-\theta_0)$ and $\cos(\theta-\theta_0)$ around $s = \theta-\theta_0$, it yields the potential:

$$f_w(X,Y) = X^4 + cs^4 + X^2s^2\cos\left(\frac{\theta_0}{2}\right) + X^2s\sin\theta_0 - Xs^2\sin\left(\frac{\theta_0}{2}\right) + 4c\theta_0s^3 - X^2\cos\theta_0 + \\ Xs\cos\theta_0 + \left(6c\theta_0^2 - d\right)s^2 + X\sin\theta_0 + 2\left(2c\theta_0^3 - d\theta_0\right)s - d\theta_0^2 + c\theta_0^4$$

For each value θ_0 of θ , the potential $f(X,\theta)$ is approximated around θ_0 by the potential $f_{w(\theta_0)}(X,Y)$ of the double cusp defined by the following values of $w(\theta_0)$:

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[19]This is the approximation obtained by taking the first terms in *s* of cos(*s*) and sin(*s*), that is by reducing their Taylor expansion to order $k \leq 3$ or *4*. This is referred to as the path's *k-jet*.

$$k = c, h = \cos\left(\frac{\theta_0}{2}\right), f = \sin\theta_0, g = -\sin\left(\frac{\theta_0}{2}\right), g' = 4c\theta_0, c = -\cos\theta_0, d = \cos\theta_0,$$

$$e = 6c\theta_0{}^2 - d, a = \sin\theta_0, b = 2\left(2c\theta_0{}^3 - d\theta_0\right), - d\theta_0{}^2 + c\theta_0{}^4$$

where g' is the coefficient of a supplementary term in s^3 and p a constant term.

This is what I meant in *H1* when I said that the *CF* can be represented by a "path" in the external space of the double cusp. I hope that this way of expressing it has now been sufficiently clarified. The path expresses the progressive internalization of the domination of the y value over the x value through the mediation of b, and then the "triumph" of b (elimination of a).

8.6. The double cusp as a classifying space

Once we have demonstrated that Pierre Maranda's model of mediation can be embedded in the double cusp, *the geometric richness of this complex singularity can be used as an underlying complexity immanent to the CF conceived of as a "structural equation"*. Many other paths can be found, which constitute so many different solutions to the problem of coupling two oppositions defined along different semantic axes. Some will be modes of internalizing the dynamical evolutions of an actantial antagonism handling an initial semantic conflict. But the study of other paths may prove equally interesting.

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