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MORPHODYNAMICS AND THE CATEGORICAL PERCEPTION OF PHONOLOGICAL UNITS

Analyzing phonetic phenomena of categorical perception enables one to understand how the perceptual treatment of linguistic sounds segment and pattern the continuous audio-acoustic flux, and how this continuum can thus be transformed into strings of discrete elements from which a code can be built. These phonetic phenomena of categorical perception are the mediating processes between phonetic substance and phonological form, and have been so far assessed in terms of two conflicting conceptualizations: on the one hand, the reductionist and 'substance-based' approach of psycho-physical phonetics, and, on the other, the morphological and 'form-based' approach of structural phonology. This article attempts to show that phonetic phenomena of categorical perception are typical perceptual cases of critical phenomena (analogous to the phenomena of phase transitions that are observed in thermodynamics), and that their bifacial reality is congruent with the general models of critical phenomena propounded by Catastrophe Theory. Therefore it makes sense to apply the latter in the analysis of the former. The ultimate goal of this article, which summarizes in part a forthcoming book, is to lay the first bases for a catastrophe theoretic phonology.

1. Categorical Perception

1.1 *Definition*

Discovered in 1957 by A. Liberman, categorical perception contrasts with continuous perception.

Let us take a 'continuum' of stimuli extending from one syllable $S_1 = C_1V$ to a syllable $S_2 = C_2V$ (with the same vowel) and such that the consonants C_1 and C_2 (for example stop consonants) do not differ in more than one single acoustic cue (for instance in voicing, such as in [ba]/[pa], [du]/[tu] etc., or in the point of articulation as in [bo]/[go], [pi]/[ti] etc.). Actually this 'continuum' is a discrete sequence of N stimuli (N being of the order of ten) the first and the last of which are natural (articulatorily producible) and the others, the intermediate, are synthetic. If one presents these "data" to a group of subjects in a sequence of identification and discrimination tests (for instance by the ABX-method) one finds *that there is no "intra-categorial" discrimination:* subjects do not distinguish between two adjacent stimuli n and n + 1 unless these are placed on one or the other side of a boundary which separates two adjacent categories. In other words, and contrary to what happens with a continuous perception as that of colours, their distinction *depends on identification* and is made in absolute terms, not in relative ones (see fig. 1). As M. Studdert-Kennedy and A. Liberman pointed out,

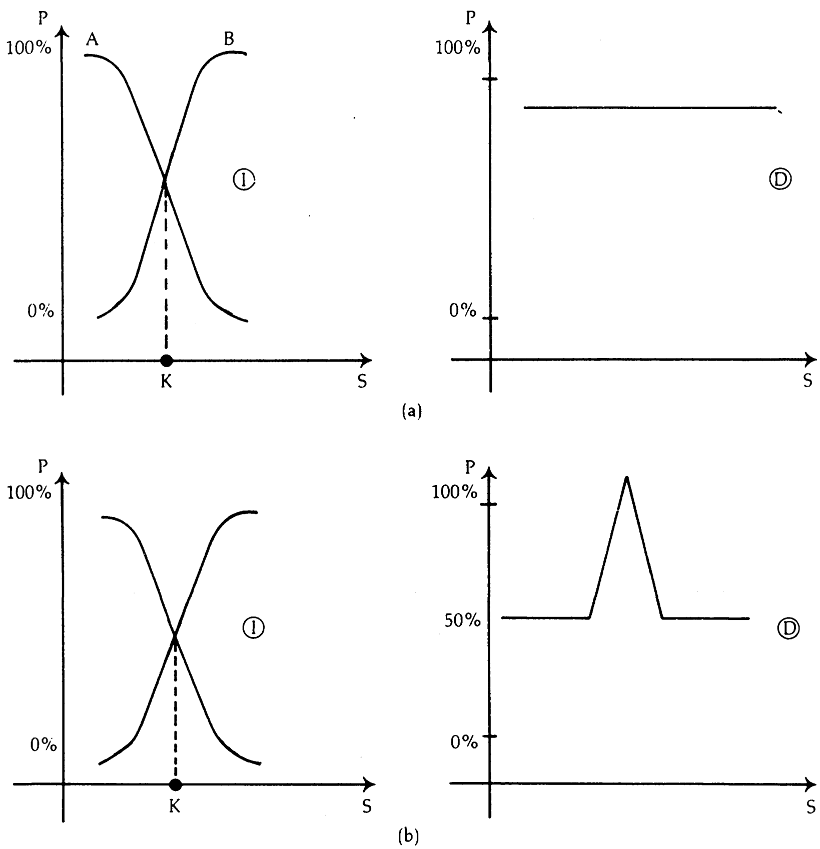


Figure 1. P = percentage S = stimuli
(a) Continuous perception (I = identification, D = discrimination, K = boundary between two categories).
(b) Categorical perception.

"categorical perception refers to a mode by which stimuli are responded to, and can only be responded to, in absolute terms" (1970: 234).

1.2 Function

The functional importance of categorical perception is evident. It is in fact the absence of intra-categorial discrimination which explains *the perceptual discretization* of the audio-acoustic flux, and why this flux can be used to support the phonological code. This discretization applies essentially to consonants (and more particularly to stop consonants), that is to say, to phonemes *encoded* in the flux (the perception of vowels and fricatives for instance, is more continuous than categorical). The encoded phonemes are categorical as produced by their specific perception devices and this fundamental fact leads to thinking that there exists a specific way (a "speech mode") of processing and decoding which is associated with phonetic perception (Liberman et al., 1967).[1]

1.3. General abstract situation

Phonetic phenomena of categorical perception are due to the fact that acoustic cues *control* the percepts. They come under the following general abstract situation. Let $(u_1, \ldots u_r)$ be parameters (viz. acoustic cues) varying over a space W, and controlling the internal states of a "black box" S (viz. a perceptual system). What needs to be understood is how such a controlled system may categorize its control space. This situation is completely different from that described by automata theory. Indeed, instead of being concerned with a discrete set of inputs and a discrete set of outputs generated by transitions between a finite set of discrete internal states, one is concerned with a *continuous* set W of inputs which function as control values. The transitions between internal states do not produce outputs but induce a system K of *interfaces*, of boundaries, of *discontinuities* in the external space W. There are cases in physics which are typical of this general situation, namely the phase transition phenomena. It is thus quite legitimate to treat (first analogically

[1] Categorical perception is very specific: if the consonantal part of a syllabic continuum C_1V-C_2V categorically perceived is extracted (transition of formants and burst noise) one will obtain a continuum of non-phonetic noises ("chirps") whose perception is *continuous*.

and then theoretically) categorical perception as an induction of phase diagrams in the acoustic cue spaces controlling the percepts.

1.4. Examples

A great number of experiments on the categorical perception have been conducted in the last twenty years. The boundaries K induced on the VOT dimension ("voice onset time" i. e. voicing cue) by the identification of the fundamental pairs of stops [b]/[p], [d]/[t], and [g]/[k] have been the object of particularly intensive study. Pioneer experiments were carried out in 1970 by Lisker and Abramson who analyzed the variation of K in function of the point of articulation (see fig. 2). These experiments, however, are still insufficient. Actually – since the point of articulation depends (as does voicing) on *continuous* acoustic cues (as for instance the frequency of burst noise of the transition of the second formant, see the locus theory of P. Delattre) – the boundary system K induced by the categorical perception categorises a *multidimensional* external space W of dimension r. Thus, since K must *classify* and *discriminate* the percepts controlled by W, it must decompose W into domains (categories). This requires *a priori* that it should be of codimension 1 (i. e. of dimension r – 1). Moreover, the fundamental information is the *geometric* information provided by its morphology. But, as it appears clearly in Fig. 2,

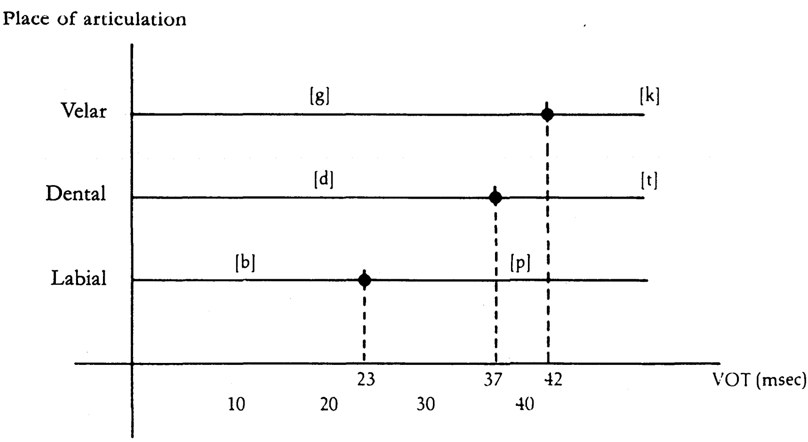


Figure 2. The experience of Lisker and Abramson are insufficient: 3 points are not sufficient to reconstruct the codimension 1 boundaries which classify the stop consonants relatively to voicing and place of articulation acoustic cues.

the results of Lisker and Abramson do not permit the reconstruction of a morphology of codimension 1 (i.e. of dimension $2 - 1 = 1$) in the external space of the VOT and of the point of articulation.

Certain attempts to explicitly reconstruct a phase diagram in an acoustic control space have been successful. The experiments of B. Repp on the English fricatives (Repp et al. 1978) are examples of this kind. Repp considers two parameters of control: an instant of silence Δ S and an instant of fricative noise Δ B. He analyzes how they are integrated in the discrimination of fricatives and affricates. In the case of an utterance such as "Did anybody see the gray ship", the external space (Δ S, Δ B) is categorized in four domains which correspond to the perceptions [gray ship], [gray chip], [great ship], and [great chip] (see fig. 3).

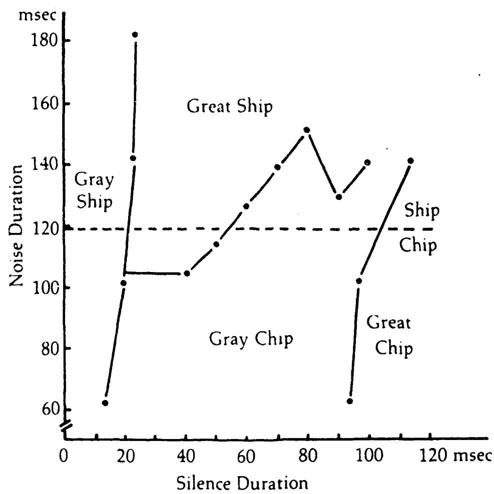


Figure 3. From Repp et al (1978). Boundaries that divide the several response categories, represented as joint functions of duration of silence and duration of fricative noise.

1.5 *Specificity*

Contrary to what was believed when research began, categorical perception is not specifically phonetic. It is found more or less ubiquitously in the whole sound domain. For instance, the perception of musical timbres is categorical (Cutting and Rosner, 1974). The same holds for perception of musical intervals by professional musicians, possessing an "absolute ear" (Siegel and Siegel, 1977). As it is known that professional musicians process musical information preferentially in the left hemisphere (the dominance of the right ear being testable through experiments of dichotic listening), one is led to suppose that there is in this hemisphere a categorical mode of perception which discretizes the information and transforms it into a code.

There also exist phenomena of categorical perception which only concern temporal organisation, thus a very abstract level. J. Mehler has shown, for example, that if one distributes three beats 1, 2 and 3 on an interval of 600 milliseconds with the second beat being situated in a somewhat intermediate variable position, the perception is categorical. It partitions the stimuli into three classes corresponding respectively to the invariant perceptions 1–2/3, 1/2/3, and 1/2–3, the boundaries being situated at about ± 20–30 ms from the central position where beat 2 is at 300 ms (Mehler and Bertoncini, 1980).

Phenomena of categorical perception do also exist in the visual domain. A typical example is supplied for intermittent luminous stimuli, by the existence of a sensorial threshold beyond which the stimuli are perceived as being continuous (threshold of "flicker-fusion").

1.6 *Innateness*

Let us come back to the categorization of the VOT dimension. If one is experimenting with languages such as English and French, where there exist no more than two possibilities for voicing (voiced or unvoiced), one will find a boundary separating for instance [d] and [t]. Experimenting with a language like Thai where there are three possibilities of voicing will establish two boundaries separating [d] from [t], and [t] from [t^h] (aspirated [t]). Experiments of this kind show that the phonological classification of a language results from the systems of boundaries K_s induced in the spaces of acoustic cues by categorical perception. These systems depend on the language (the boundary [b] / [p] on the VOT axis for instance is at 37 ms. in English and at 5 ms. in French) and this fact confirms the *relativity* of phonological categories. It seems, however, that the systems K_s which are part of the steady state of the language, all derive from an *innate* initial state K_0.[2]

[2] For the concept of an innate initial state, see the Chomsky-Piaget debate (Piatelli-Palmarini 1979)

Actually, a certain number of concordant experiments conducted in early (pre-verbal) childhood prior to language acquisition seem to show that there is a universal VOT categorization (genetically determined), defined by two boundaries, one at approximately -30, -20 ms. and the other at approximately $+20$, $+30$ ms. (see fig. 4).

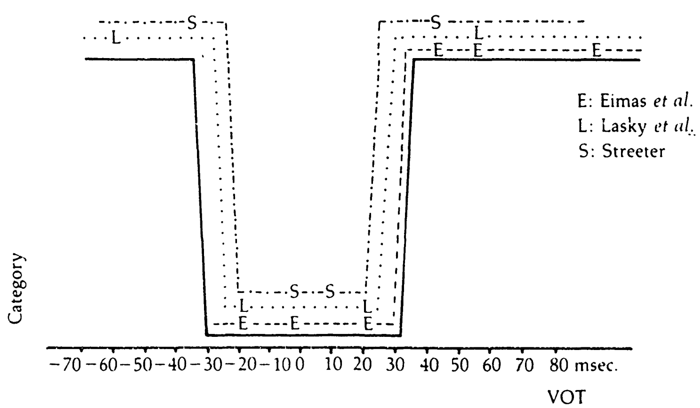


Figure 4. The universal VOT categorization for preverbal infants. From Mehler-Bertoncini (1980). Lasky (1975) deals with spanish infants and Streeter (1976) with Kikuyu infants.

The existence of an innate sensorial component, although it refutes the old idea of "tabula rasa", is a prerequisite for the acquisition of language. For, as P. D. Eimas (1980) observed, to be able to learn its first language, apart from knowing the intrinsic organization of the basic units, e.g. phoneme and syllable, a child must know:

i) how to discriminate small differences in the acoustic signal.
ii) how to categorize a continuous sequence of acoustic values;
iii) the intrinsic organization of the basic units, phonemes and syllables;
iv) how to arrive at invariant perception despite the variations of the signal;
iv) how to process in a context-dependent mode, the acoustic information which is critical for phonetic distinctions.

2. The conflicting Interpretations

There are several interpretations of categorical perception. They differ in terms of the classical oppositions sensorial/cognitive and reductionist/structural. Their conflict is of special epistemological interest since it shows that, without deep conceptual and theoretical considerations, it is quite impossible to decide experimentally between the various paradigmas.

2.1 The Sensorial Hypothesis

Some authors have advanced the hypothesis that categorical perception is a *general* property of perception, to be deduced from psychophysical principles. Thus, according to J. Miller (Miller et al., 1976) it indicates the presence of a masked sensorial threshold. In all experiments, one considers a continuum associated with the variation of a cue I and discretized in equal steps Δ I. Below the threshold, the variations Δ I will not be detectable (and thus not discriminated). Above the threshold they will be also not discriminated by virtue of Weber-Fechner's law.

This hypothesis was taken up again by R. Pastore in order to account for the categorical perception without subordinating sensorial discrimination to cognitive identification. According to Pastore, the origin of categorical perception is to be found in the structure of peripheral neuroperceptual processes and depends upon a limitation, may it be internal or external, which is at the same time stable and defined more precisely than the threshold of discrimination. The limitation is internal if it corresponds to a masked threshold; it is external if it involves a stimulus of reference with which the stimuli are to be compared (Pastore et al., 1977).

2.2 The Reductionist Hypothesis of Feature Detectors

The experiments with prelinguistic children question the classical theories of phonetic perception, which treat it as an *articulatorily* finalized process; (i. e. one which is realized by way of 'mental reconstruction' of the articulatory process).

Actually, with the infants perceiving before articulating, one has in fact to assume that there is a genetic determination of the feedback perception → articulation. But "to attribute this knowledge to the infant's biological endowment would seem to extend considerably the cognitive competences that we are willing to impute to genetically determined factors." (Eimas 1974: 518). On the other hand, experiments on dichotic hearing seem to indicate that perception works by recombining on the central level, the distinctive features extracted from the acoustic signal at the peripheral level. Finally, since they were discovered by Eimas and Corbitt in 1973 one has at hand a number of concordant experiments concerning the phenomena of *selective adaptation* (Eimas and Corbitt, 1973).

In the selective adaptation experiments one considers a continuum W (for example [ba] – [pa]) and repeatedly presents one of the bounding stimuli to the subjects. If now after this adaptation one constructs the boundary K categorizing W one can see that, with respect to the situation without adaptation, it has *shifted* towards the adapting stimulus. If the adapting stimulus is now no more one of the series tested, but a different one which bears no relation to the former other than a common distinctive feature, an identical shift will be observed (adaptational selectivity).

These results have lead Eimas and Corbitt to set up the hypothesis of *feature detectors*, which are neuro-sensorial receptors responding selectively to well-defined domains of acoustic cue values. "'Feature detectors" can be broadly defined as organizational configurations of the sensory nervous system that are highly sensitive to certain parameters of complex stimuli.' (Abbs and Sussman, 1971: 24). Supposing that they are part of the initial phonetic state of the organism, that they are only sensitive to very limited domains of variations of acoustic cues and that their domain of response is acquired through interaction with the environment, one can easily explain the previous experiments. In order to do this, it is sufficient to suppose:

i) that the VOT axis is covered by the domains of two (or three) peripheral detectors;
ii) that the responses of these detectors are *in competition* and that the central systems of information processing are only sensitive to the detector which gives maximal response;
iii) that repeated presentation of the same feature 'tires' the corresponding detector (see fig. 5).

The reductionist hypothesis of feature detectors has had great impact not only because it has provided a simple explanation of categorical perception and of phenomena of selective adaptation, but also because it has applied in phonetics a perspective which, since the work of Hubel and Wiesel, is well known in visual perception. Thus it leads to a unified neuro-physiological conception of perception. However, because it has a reductionist character, it does not work without theoretical problems.

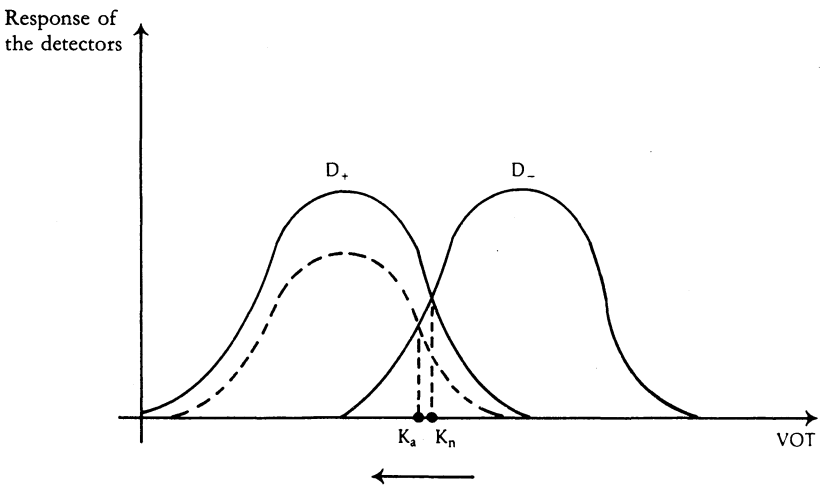


Figure 5. The feature detectors explanation of the boundary shift. The VOT axis is covered by the domains of two detectors D_+ (voiced) and D_- (non voiced). Selective adaptation by a voiced stimulus makes the D_+ response decrease and the normal boundary K_n shift to K_a.

2.3 Criticism of the Feature Detectors Hypothesis

The feature detectors hypothesis has been criticized, not only by the supporters of the motor theories, to which it is opposed, but also by those who support theories which are reductionist as well. First, as R. L. Diehl has remarked, one can explain the experiments of selective adaptation using other perceptive principles, as for example the contrast principle according to which the perception favours difference against identity. If the adaptation serves as reference, then the tested stimuli of the continuum will be perceived much earlier as different and the boundary will then be shifted to the adaptator (Diehl et al., 1978). On the other hand, as was noted by J. S. Bryant, the effects of selective adaptation simply show that the phonological distinctive features are neurologically represented. "This does not necessarily mean that the sum of such representations is functional in perception as the feature detector notion implies. Rather, the cell or cells may simply be responding as a small part in a large pattern of neural response to the stimulus." (J. S. Bryant, 1978).

We do criticize for three reasons this hypothesis of feature detectors. Firstly, since simple relations between acoustic cues and encoded phonemes do not exist, it necessarily leads to a *proliferation* of detectors. Furthermore, the interface induced by categorical perception on an axis as that of the VOT, *varies* with other cues (for instance, the point of articulation). But this does not agree with the hypothesis of *independent* feature detectors. The spaces of cues W that are associated in a natural way with the phonemic "Gestalten" are *multidimensional*, and it is the *morphology* of the categorizing systems K which is important.

In dimension 1, the theoretical difficulty posed by the generation of the K's, does not appear because – whatever the imagined generating mechanism may be – the K's reduce to isolated points. However, *this is not the case in higher dimensions*, where the morphology of boundaries provides a simple criterion for the falsification of the existence of detectors. Indeed, let us suppose that a two dimensional space W of cues is covered by the domains of response of a finite number of detectors whose surfaces of response are bell-shaped (by analogy with the one-dimensional case). Then K must be the projection onto W of the intersection curves of these surfaces.

Still, the morphologies thus obtained are very different from the generic morphologies of interfaces that one find in phase transitions phenomena and, more generally, in critical phenomena. So, if one could show (and it will be very important to do experiments) that the observable morphologies of phonetic *multidimensional* boundaries are of the second and not of the first kind, one will be able to falsify the hypothesis of feature detectors, at least in its naive version.

But our principal criticism against this hypothesis is that it is dogmatically reductionist and marks a theoretical regression with respect to structural advances. Inspired by specialists of information processing, it favours, as has been emphasized by Bryant, recognition against perception and memory-stacking structures against processes. Considering as evident the reduction of the global neuronal dynamics to the structure of the underlying neural net, it belongs to the computational paradigmas which reduces perceptual processes to logical (propositional) calculus (see for instance Miller and Johnson-Laird, 1976).

2.4 *The Fundamental 'Antinomy' of Phonetics*

The debate around the sensorial vs. cognitive, the reductionist vs. the structural interpretations of categorical perception is of great epistemological interest in so far as it represents the actual state of a difficulty which one could call the fundamental 'Antinomy' of phonetics. It concerns the possibility of explaining on a *psychophysical* basis the phonemes as abstract, *linguistically functional*, units. These units are not defined by intrinsic properties but by a network of differences. Since the day of Saussure and Jakobson, the phonemes have actually been viewed as purely *relational* entities (and not substantial ones), as "Gestalten" depending upon a *formal* level of reality. We are therefore committed to treat *two* categorialities (in the epistemological sense of "categoriality"):

i) On the phonological side, a structural categoriality (Jakobsonian-Hjelmslevian) that is believed to be of a logico-combinatorial kind: identity of position, relational unity, difference, discrimination, reciprocal determination, stratification, etc.
ii) On the phonetic side, a psychophysical categoriality: spectral forms, deformation of these forms, control by acoustic cues, invariance/variability, categorization, boundaries, etc.

The problem is evidently to *unify* these two categorialities. Now this is impossible if one interprets the first one in a logico-combinatorial way (i. e. formal in the formalist sense of the term) and the second in a reductionist physicalist way. Thus we are faced with a real antinomy reminiscent of the Kantian antinomies. The hypothesis of feature detectors "dogmatically" solves this antinomy by annulling the structural categoriality in favour of the psychophysical one. But this is a misleading point of view. For the real problem is actually to *constitute* (and not to deny) the objective value of the structural categoriality. We face here a deep *mathematical* problem of 'schematization' (in an actualized Kantian sense) of the structural categories.

Thus, if it is legitimate to demand that "at all levels of the hierarchy of language and for every category of linguistic elements, one must suppose an abstract and functional aspect, uniquely described in relational terms, and a substantial aspect, which is described, according to the choice and the aim of the person describing in terms of articulation, acoustic structure, and auditive perception," (Malmberg, 1974: 210) one must still understand the relation of reciprocal dependence between the *phonemic form* and the *phonetic substance*. Without such an understanding, the only alternative to the notoriously unsatisfactory reductionist conceptions is a structural realism which is equally unsatisfactory in so far as it naturalizes the formal artefacts and finally calls for a recourse to improvable innatist hypotheses.

2.5 *The Paradigmatic Categorization a priori*

In order to understand the relation of reciprocal dependence between form and substance, one has to define the general mathematical content of the abstract situation described in § 1.3 and show:

i) that this content allows to schematize in a unifying way the structural and the psychophysical categorialities.
ii) that, by specification of its general content, this schematization yields models fitting the experimental data.

In other words, before trying to work out specific models, one has to formulate mathematically the *a priori* of categorical perception, that is to say the *a priori* of paradigmatic categorization.

Here, the leading idea is that the concept of categorisation is the synthesis of the ancient concept of *classification* and the modern concept of *control. A priori*, the general abstract situation described in § 1.3 is of the following type. Let W be a space of acoustic cues (which one supposes to be reduced to simple parameters) $u_1, \ldots u_r$. A point $w = (u_1, \ldots u_r) \in W$ is then an input stimulus for a 'black box' S. In this 'black box' a global dynamical process X (which may be conceived as being reducible to a neurophysiological determinism) defines internal states A, B, C... (acoustical percepts) and is controlled by W. Moreover an instance I selects, for a given $w \in W$, the *actual* internal state, the other internal states being virtual. The general situation is thus characterized:

i) by a field X_w of dynamical processes controlled by W, i.e. a mapping $\sigma : W \to \mathscr{H}$ that associates with each point $w \in W$ a 'point' X_w of the generalized space (functional space) $\mathscr{H}$ consisting of all possible internal processes;
ii) by the selection instance I.

To understand how such a system $S = (W, \mathscr{H}, \sigma, I)$ can categorize its control space, it is sufficient to postulate that the perception is not determined by the exact form of the actual internal state A but only by its *qualitative type* τ (A). This qualitative type will generally be defined by the action of a group G on the functional space $\mathscr{H}$. If X is a point of $\mathscr{H}$, its orbit $\tilde{X}$, under the action of G is constituted by qualitatively equivalent processes which define the internal states of the same qualitative type. One can try to characterise these by the values of a system of invariants $\tau_1, \ldots, \tau_k$, i.e. by 'properties' of the associated phonemic percepts.

On the space $\mathscr{H}$ there exists generally a topology (even several), which allows us to say when two processes $X_1, X_2 \in \mathscr{H}$ are 'neighbours'. Now the existence of such a topology and the action of G on $\mathscr{H}$ are sufficient to determine the *structural stability* of the elements of $\mathscr{H}$: We say that $X \in \mathscr{H}$ is structurally stable if and only if all $Y \in \mathscr{H}$ sufficiently close to X are G-equivalent to X. Now let $K_{\mathscr{H}}$ be the subset of $\mathscr{H}$ consisting of all processes that are structurally *instable*. $K_{\mathscr{H}}$ is a *discriminant* morphology, inherent in $\mathscr{H}$, which *categorizes* it and *classifies* the qualitative types of its stable elements. In other words, every space $\mathscr{H}$ of processes (and more generally, every space of forms) where one can define the notions of deformation and of qualitative type can be categorized in a natural way through a subset $K_{\mathscr{H}}$ called its catastrophic subset and which gives a *geometric* meaning to the classification in $\mathscr{H}$. In other words again, once mathematically interpreted (in a very general way though), in terms of the action of groups on generalized spaces (and no longer in a logical-set theoretic way) the classical concept of classification gives rise to a new geometry and it is this geometry which constitutes the paradigmatic categorization *a priori*.

For let $W \xrightarrow{\sigma} \mathscr{H}$ be the field expressing the control of the system S by the control-space W. σ maps W into $\mathscr{H}$ and it may be supposed that this mapping itself is structurally stable, which imposes drastic restrictions on its complexity. Now let $K' = \sigma^{-1}(K_{\mathscr{H}} \cap \sigma(W))$ be the σ-preimage of $K_{\mathscr{H}}$ in W. The categorization of W induced by S and defined by the system of discontinuities K is deducible from K′ if one knows I. Actually, the dynamical origin of K is the following: Let A_w be the actual state chosen by I at $w \in W$. If w varies in W, A_w – if supposed to be stable – does not change its qualitative type, which implies the invariance of the associated percept and of the properties $\tau_1, \ldots, \tau_k$. However, in general there do exist critical values w_i of w for which A_w comes into conflict with another internal state B_w and is supplanted by it after the crossing of w_i. Thus K consists of these critical values corresponding to a catastrophic transition of the actual state. Now these catastrophes will be correlated in a regular fashion by I, either with an internal destabilization of A_w (bifurcation catastrophes) or with the fact that A_w comes into conflict with B_w for intrinsic reasons (conflict catastrophes). But for a process X, a destabilization of an internal state or a conflict between two internal states are causes of instability and w belongs to K if and only if the situation at w is correlated in a regular fashion by I with a situation which belongs to K′. In this sense K is deducible from K′. So the paradigmatic categorization *a priori* is: the categorization is the trace in the control-space of the instabilities and conflicts of the internal states that it controls.

2.6 *Elements of Structural Criticism*

The clarification of the catastrophical categorization *a priori* allows us to develop, in a fashion very close to the Kantian one, the "Elements" of a 'Structural Criticism' (Petitot 1982, 1985). Let us give some indications.

2.6.1 Internal discontinuous features and external continuous features

Given that cues ($u_1 \ldots u_r$) control properties of percepts ($\tau_1 \ldots \tau_k$) which are invariants of their qualitative type, the notion of 'distinctive feature' must be reconsidered. One has to distinguish between the *external* features (which are the cues) and the *internal* features (which are the qualitative invariants). The former vary in a continuous way, whereas the latter vary discretely, and this necessarily according to the two phenomenological types of opposition, emphasized by Jakobson:

i) the *qualitative* oppositions corresponding to the conflict catastrophes (competition of two invariants),

and

ii) the *privative* oppositions corresponding to the bifurcation catastrophes (presence/absence of a unique invariant).

In the case of voicing, for instance, the external feature is provided by the VOT, categorized by categorical perception, and the internal feature is provided by the privative opposition voiced – unvoiced.

In the new theoretical framework, founded on the '*a priori*', one is able to overcome the antinomy between the 'continuous' and the 'discontinuous' and to legitimize the Jakobsonian binarism on the very basis of continuous variation of the cues. This solves one of the major difficulties of phonetic descriptions: "one of the major difficulties in achieving this kind of description is in relating the essentially continuous nature of speech with the essentially discontinuous nature of linguistic description." (Ladefoged 1972: 276; see also Massaro 1972).

But there are two other difficulties. First of all, the external features *do not* generally coincide with the cues. For let W be a space of cues ($u_1, \ldots, u_r$). The essential information is provided by the discriminating morphology K (the catastrophic set) induced into W by the perceptual 'black box'. *A priori* there is no reason at all that the coordinate system (the frame) R of W constituted by the axes $u_1, \ldots, u_r$ should be 'adapted' to K. The position of K relative to R affords the fundamental data about the non-independence (and therefore about the *integration*) of the cues. It is therefore natural to look for a coordinate system R of W adapted to K. It is this frame to which the external features correspond. This, moreover, permits us to understand why these – although being continuous – can be generally *discretized* and reduced to an opposition of the type +/– (which is not to be confused with qualitative or privative oppositions describing the internal features). Indeed one has to localize not so much the *points* of W as the *domains* of W which K differentiates, classifies and positions, one with respect to the other. This can generally be done with a discrete information. Let us assume for example that W has dimension 2 (cues x and y) and that K consists of two curves γ_1 and γ_2, which intersect transversally in w_0. K then divides W into four domains and in order to position these domains *locally*, it suffices to consider the adapted coordinate system, constituted by the corresponding tangents T_1 and T_2 at γ_1 and γ_2 in w_0. Each domain becomes 'coded' by a half-tangent (+ or –) at γ_1 and a half-tangent (+ or –) at γ_2 (see Fig. 6). It follows that, as far as the *external* features are concerned, the Jakobsonian binarism (*a priori* valid for the internal features) has to be interpreted as an information about *the type of local complexity of phonetic boundaries:* these are sufficiently simple such that their adapted coordinate systems can be discretized in a binary way. And the fact that the combinatorics of distinctive features is a *constrained* one, just as the phenomena of stratification of features (i. e. the relations of marking and dominance) give informations as well about what separates the local structure of K from that described in Fig. 6 (which is a case of free combinatorics) as about its global structure.

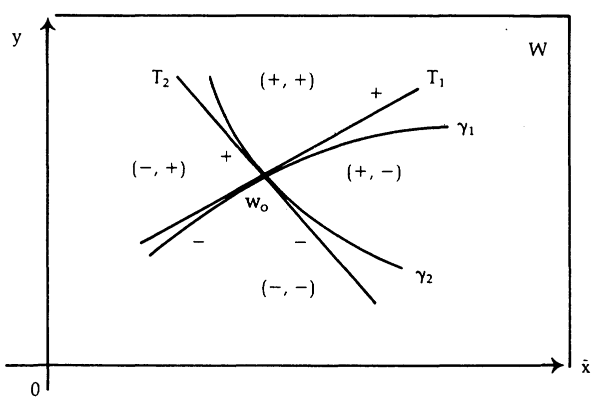


Figure 6. The discretization of a frame adapted to $K = \gamma_1 \cup \gamma_2$. Every domain of W defined by K is coded by a pair (±, ±).

The second reason why the elementary catastrophic models provide only the categorical perception *a priori* and are not exact models is that the phonetic system has *two* levels of control. On a first level, a space A of articulatory parameters controls continuous spectra belonging to a spectral space S. One has thus a first field $\alpha : A \to S$, possessing the important advantage of being observable and corresponding to the variation of the spectrograms relative to articulation (see the classic works of P. Delattre at the Haskins Laboratories). Yet on a second level, the auditive transformations of the acoustic spectra, form a space F which controls a space P of mental processes, defining the percepts. Hence one obtains a second field $\sigma : F \to P$, not observable directly and whose space of control is not a space of parameters but a space of forms. The two levels are connected on the one hand by the auditive transformation: $T : S \to F$ and on the other hand by a *feed-back* $\varphi : P \to A$, expressing that perception works through the reconstruction of an articulatory motor-scheme and that, correspondingly, articulation is controled by a space of internalized phonemic targets. In this general frame

$$\begin{tikzcd} A \arrow[r, "\alpha"] & S \arrow[r, "T"] & T \arrow[r, "\sigma"] & P \arrow[lll, bend left, "\varphi"] \end{tikzcd}$$

the distinctive features can be conceived:

i) as coordinate systems adapted to the boundaries induced into A be it by S, be it by F, be it by P; i.e. like external features of articulatory nature, which would be the trace of instabilities and conflicts of invariants of the acoustic spectra, of their auditive transforms or of their acoustic images;

ii) as properties of qualitative types of the acoustic spectra; i.e. like internal features of acoustic-auditive nature, characterizing elements of S or F;

iii) as coordinate systems adapted to boundaries induced into S or F by the perception P; i.e. like external features of acoustic-auditive nature which would be the trace of instabilities or conflicts of 'attractors' of neurological dynamics; and finally

iv) as invariants of these 'attractors'; i.e. like internal features of perceptive nature.

2.6.2 The Discretization Condition

The categorization *a priori* allows to overcome the antinomy of the discrete and the continuous, and to unify the two conceptions that one can have of a phoneme:

i) a "Gestalt" serving as a prototype for a class of allophones (logical-combinatorial criterion for the identity of a phoneme)
ii) a domain in a control space (positional-structural criterion for the identity of a phoneme).

As prototypic "Gestalten" the phonemes are as the 'capitals' of the domains defined by the categorization (W, K).

Moreover the *a priori* makes appear *an irreducible conflict* between the logical-combinatorial criterion for the identity and the positional-structural one. Indeed, in order that these criteria could be compatible, a condition which we call the *discretization condition* has to be fulfilled. It says that there exists a one-to-one correspondence between the prototypes and the connected components of the complement W−K of K in W. It is fulfilled for the categorization shown in Fig. 6 (see fig. 7). Yet, there is no reason at all why it should be fulfilled *a priori*. For example, let us consider, in a control-space W of dimension 2, an interface K, halting at a point δ. K is a threshold, separating two determinations A and B according to a qualitative opposition A/B. Now at δ, the threshold disappears and a neutral-complex term A*B is generated. Hence, there exist *three* prototypes for *a single* connected component. An the fact is that this type of interfaces *cannot* be eliminated from the theory. In the physical case of phase-transitions (which, as we have seen,

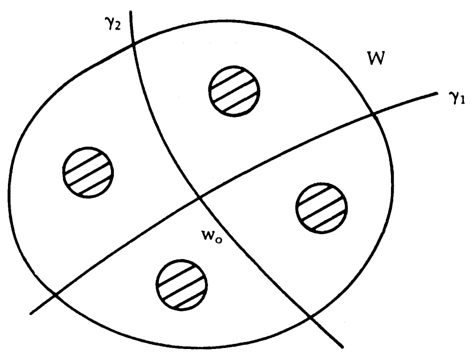
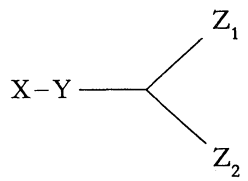
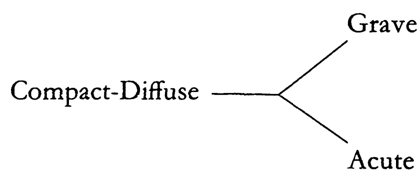


Figure 7. The discreteness condition. There is a bijective correspondence between the "capitals" and the connected components of W−K ($K = \gamma_1 \cup \gamma_2$).

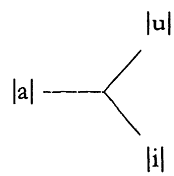
have to become paradigmatic for the understanding and explanation of categorical perception phenomena), it corresponds to the fundamental phenomenon of the existence of critical points (stop of the interface liquid/gas). In phonetics it corresponds exactly to the descriptions in terms of distinctive features of the type, that is to say to the subordination of an



opposition Z_1/Z_2 to one of the terms (Y) of a dominant opposition X/Y. This



is in particular the case *for the elementary hierarchy which classifies the universal triangle of the cardinal vowels* (see fig. 8):



This is a general and fundamental phenomenon. The determination of positional identities by systems of differences may come into conflict with the associated discrete units and violate the logical principle of identity. The positional identities do not necessarily correspond to isolable identities and to objective discrete units. In this sense, the categorization *a priori* legitimizes the structural principle (Saussurian-Jakobsonian) of ontological primacy of difference over identity. It shows that the *eidetic* content of structural categories *cannot* be of a logico-combinatorial nature.

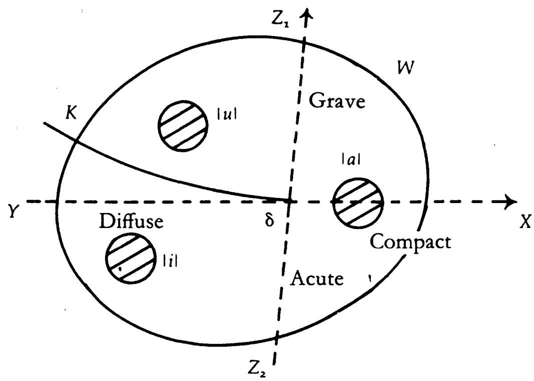


Figure 8. A case where the discreteness condition is not satisfied. The boundary K ends at the "critical point" δ. The principal axis of the adapted frame is divided in two domains X and Y (Y corresponds to K). A second axis Z_1/Z_2 is subordinated to X/Y. The adapted frame is then of type $X\text{—}Y\!<\!\begin{smallmatrix}Z_1\\Z_2\end{smallmatrix}$. There exist three "capitals", respectively coded by (–, 0), (+, +) and (+, –). But there exists only one connected component of W–K.

2.6.3 The Principle of Phenomenological Abduction

On a more profound and more general level, the catastrophic categorization *a priori* allows to unify the two conceptions of phonetics, which until now have been rivals, the 'substance-based' (reductionist) one and the 'form-based' (structural) one. The leading idea (which will become evident, we hope, by what we have already presented) is that *the phonemic form is equivalent to the organization of the phonetic substance*. In this statement, it is the term 'organisation' that is essential. As we have noted in 2.4., as long as one understands substance in a reductionist (psycho-physical) manner and form in a formalist manner, it is impossible to achieve a unified phonetic-phonological theory. In order to achieve this, one has to base the theory on the categorical perception *a priori*, and

i) understand the relational form as a logico-combinatorial description of the discriminant morphologies (W, K), i. e. of the pure phenomenology of categorizations and

ii) suppose, purely implicitly, a suitable generating mechanism X (an unobservable psychological process) that generates these morphologies.

This second hypothesis will perhaps seem difficult to admit. It is however inevitable, for the brain dynamic X is a 'black box'. The problem is not its relevance but its operational range. Reductionism is characterized by the optimistic belief that the progress in neurophysiology will one day makes it possible to explain the mechanism X causally. Yet, this is a naivety. Indeed – and it is *here* that catastrophe theory becomes epistemologically and methodologically essential – a highly developed *mathematical* analysis of the situations of control $\sigma : W \rightarrow \mathscr{H}$ constituting the categorization *a priori* shows

i) that the generating mechanisms X suitable to generate a morphology (W, K) are highly *overdetermined* with respect to the geometry of K;
ii) that in this sense the morphologies are to a large extent *independent of the substrata* and subordinated to purely *geometrical* restrictions, of formal – and not material – nature; and finally
iii) that it is possible to retrieve from the phenomenology of K constraints on X, and that in the simplest cases, one can even abduce from K a minimal model X_0 of the generating mechanism of which X is – by necessity – a complexification.

We call the latter principle *the principle of phenomenological abduction.* It reverses the order of physical deduction, which reductionism considers to be evident. Indeed, instead of trying to causally deduce K from an explicit (neurophysiological) understanding of X, one tries on the contrary in applying this principle to abduce from K (i.e. from the phenomenal manifestation) a partial knowledge of the implicit X. The two essential achievements of catastrophe theory, viz. (1) independence from the substratum and (2) the phenomenological abduction, allow us to understand how, in so far as it is equivalent to the organization of the phonetic substance, the phonemic form can be *ontologically autonomous.*

2.7 *The Precatastrophic Interpretations of Categorical Perception*

In trying to isolate the eidetic content of categorical perception, a number of authors have arrived at conceptions quite close to its catastrophic *a priori.*

2.7.1 The Prototype Models

Given a phonetic categorization (W, K), the allophones located in a neighbourhood of K are, in general, not naturally producible and have to be synthezised. The naturally produced allophones cluster in zones (like 'capitals') corresponding to *typical* values of acoustic cues. Indeed, as emphasized by Lindblom, if the articulatory production of speech is 'output-oriented' (i.e. finalized by perceptive targets), these targets are chosen in such a way as to maximize acoustic stability relative to the articulatory variability. This is one of the essential characteristic features of what Lindblom calls the 'distinctiveness condition', conceived of as 'evolutionary conspiracy' (Lindblom, 1972). P. Ladefoged has introduced as well the notion of 'conventions of interpretation', associating the ideal form of a discrete linguistic unit with the typical values of the control-parameters (Ladefoged, 1972).

According to this point of view – which D. Massaro has termed 'the template matching scheme' (Massaro, 1972) – the phonemes are conceived of as unitary "Gestalten". It is assumed that in the space P of percepts (which is supposed to be equipped with a perceptive distance) there exist finitely many prototypical phonetic patterns $p_1, \ldots, p_n$. Given a percept p, a device tries to minimize the distance from p to the p_i and recognizes p as a token of the type p_i (as an allophone of the phoneme p_i) whose distance to p is minimal. The categorization K is thus composed of a set of median lines. We call this a T-classification (see fig. 9). Such a model has been developed by Repp.

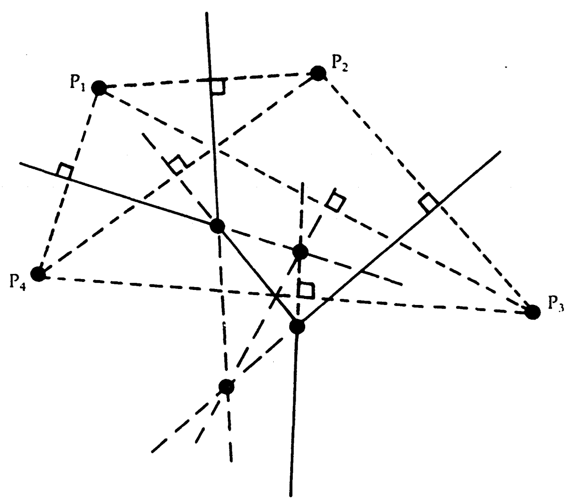


Figure 9. An exemple of a T-classification. Let be given n prototypes $p_1, \ldots, p_n$. The domain of a prototype p_i is the set of points p for which the distance $d(p, p_i)$ is less than $d(p, p_j)$ $j \neq i$.

2.7.2 The Precatastrophic Models

Along with the prototype-models associated with T-classifications, models of a precatastrophic nature were developed in association with K-classifications (but without adequate mathematics). In these models, which formulate the categorization *a priori* $\sigma : W \rightarrow \mathcal{H}$, one starts from the following hypotheses:

i) the *continuous* acoustic spectra (the harmonic spectra are not, as one knows, phonologically relevant) are controlled deformable forms;
ii) audition extracts *qualitative* information from these spectra;
iii) some types of spectra are *stable* relative to their control, and others are not;
iv) perception retains only invariants from qualitative spectral types;
v) the phenomena of categorical perception are due to the catastrophic transitions of these qualitative types under the action of control. As noted by Pisoni, there can be categorical perception wherever complex entities controlled by parameters contain several distinctive qualities whose presence or absence defines domains of control; these categories can be labeled and coded in the short-term memory (Pisoni, 1979).

The first leading idea now is that one has to treat the percepts 'not as bundles of separately extracted phonetic features but as integral multidimensional entities whose dimensions are inseparable aspects of the whole pattern.' 'The dimensions are assumed to reflect the auditory properties of the stimulus and thus are continuous, not binary. Instead of representing speech sounds as matrices of discrete feature values, they are conceptualized as points in a continuous multidimensional perceptual space.' (Repp et al., 1978). Thus, one no longer postulates that perception is based on a detection of cues combined afterwards in a unified percept, but one starts from the *a priori* control and imagines that 'the sound simply initiates the unfolding for a complex pattern of neural response that directly supports the phenomenal experience' (Bryant, 1978: 616). This structural (holistic) point of view makes it possible to understand categorical perception without subordinating the sensorial discrimination to a cognitive identification. Actually, only if one opts for a model of prototypes, the identification is cognitive since it results from a comparison with data stored in the long-term memory. If one does admit on the contrary that it is based on the value of qualitative invariants, it is equivalent to a categorical discrimination without any subordination of the sensorial to the cognitive. On the other hand, in order to understand in this context the existence of innate capacities of discrimination, categorization, and perceptual invariance in pre-linguistic children, it is sufficient to admit that there is a genetically determined neuronal wiring, including an innate field $\sigma_0 : F \to P$ in the initial state of the perceiving 'black box'.

This hypothesis is perfectly compatible with Eimas' statement that 'the fact that constancy is present in prelinguistic infants is strong evidence that the means by which it is achieved is a result of biological constraints on the infant's perceptual system, such that the system must either be innately attuned to the invariance in the signal or innately able to impose constancy on acoustic diversity.' (Eimas, 1980). Yet, it does not imply at all (as it does according to Eimas) the existence of detectors. One can rather agree with Jusczyk that the initial field $\sigma_0 : F \to P$ – which should be a psychophysical universal of our species, phylogenetically inherited – determines the properties of perceptual *saliency* which can be linguistically relevant. 'As the child begins to aquire the phonological structure of the language, one would expect to see him weigh the various acoustic cues present in the speech signal according to their salience in marking distinctive contrasts in the language.' (Jusczyk, 1980).

During language acquisition the universal K-classification K_0 determined by σ_0 will be deformed, complexified and specified by the linguistic environment and progressively subordinated to a T-classification of cognitive nature.

Yet, undisputably, it is Kenneth Stevens who has come closest to the catastrophic *a priori* of the categorical perception, especially in his classic article *The Quantal Nature of Speech* (Stevens, 1972). Stevens, in considering the first level of control defined by the field $A \xrightarrow{\alpha} S \xrightarrow{T} F$ introduces two leading ideas.

i) The relation established between A and S (i.e. between the articulatory level and the (audio) acoustic level) by the control α is 'nonlinear'. There are domains of A where the associated spectra are stable with respect to α and these domains constitute the basis of the phonological code. 'For a particular range of an articulatory parameter, the acoustic output from the vocal tract seems to have a distinctive attribute that is significantly different from the acoustic attributes for some other region of the articulatory parameter. Within this range of articulation, the acoustic attribute is relatively insensitive to perturbations in the position of the relevant articulatory structure.' (Stevens and Perbell, 1977; see also MacNeilage, 1979).

ii) The acoustic attributes (whose stability and invariance with respect to the variations of articulatory control one is interested in) are spectral configurations *of higher order*, i. e. *qualitative and global* properties of the spectra and not isolated elementary cues. In other words, they are properties of the 'gross shape' of the spectra or, more precisely, of their auditive transforms. (Stevens and Blumstein, 1978).

Hence, according to Stevens, the relation between an articulatory parameter and the acoustic attribute which it controls, is typically of the catastrophic kind, as is shown in figure 10, taken from Stevens (1972). 'There are certain conditions for which a small change in some parameter describing the articulation gives rise to an apparently large change in the acoustic characteristics of the output, there are other conditions for which substantial perturbations of certain aspects of the articulation produce negligible changes in the characteristics of the acoustic signal' (Stevens, 1972: 52). This makes it possible to understand the acoustic-articulatory basis of the phonological form avoiding physical reductionism as well as structural realism (see 2.6.3).

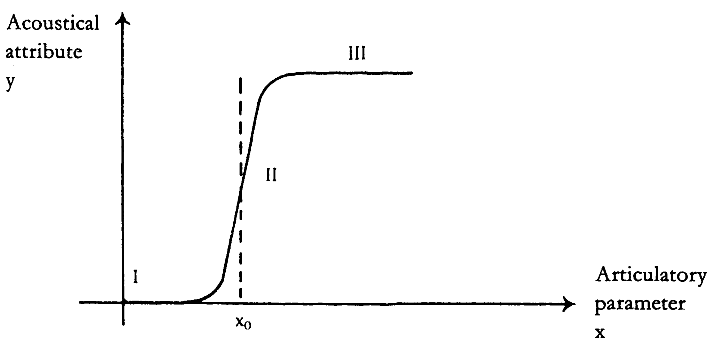


Figure 10. A typical relation between an articulatory parameter x and an acoustical attribute y. In the domain I, y is absent. In the domain III, y is present. In the domain II, y is unstable relatively to variations of x and there exists a catastrophic transition at x_0.

3. The Principles of Catastrophic Modelling

Catastrophe theory intends to study the *a priori* $\sigma : W \rightarrow \mathcal{H}$ in all cases where the processes considered are defined by dynamical systems (vector fields) X_w on differentiable manifolds M. As this project is too complex (the structure of generic dynamical systems is by itself a tremendous problem) it is first restricted, in the so-called 'elementary' theory, to dynamical systems X_w derived from potentials $f_w : M \to \mathbb{R}$ ($\mathbb{R}$ being the field of real numbers). For a given n-dimensional manifold M (assumed to be compact) one has thus to understand the structure (i) of the space $\mathcal{F}$ of differentiable functions $f : M \to \mathbb{R}$ and (ii) of the fields $\sigma : W \to \mathcal{F}$ mapping the control-spaces W in $\mathcal{F}$. The theoretical strategy is to focus on the *geometry* of the catastrophic sets $K_{\mathcal{F}}$ and K. Let us very briefly present some leading ideas.[3]

3.1 *The Qualitative Type*

The qualitative type of a potential $f : M \to \mathbb{R}$ is now its *differentiable* type, defined by the action upon $\mathcal{F}$ of the group $G = G_M \times G_{\mathbb{R}}$ where G_M (resp. $G_{\mathbb{R}}$) is the group of diffeomorphisms (i. e. of automorphisms for the differentiable structure) of M (resp. of $\mathbb{R}$). If $f \in \mathcal{F}$, its orbit $\tilde{f}$ under the action of G consists of all $g = \psi f \varphi^{-1}$ where $\varphi \in G_M$ and $\psi \in G_{\mathbb{R}}$.
If $f \in \mathcal{F}$, the essential qualitative information (G-invariant) on its structure is provided by its *critical* elements. Let $x \in M$ and let $D_f(x)$ be the linear tangent map of f at x. $D_f(x)$ maps the tangent vector space T_xM of M at x into the tangent vector space $T_{f(x)} \mathbb{R} \cong \mathbb{R}$ of $\mathbb{R}$ at f(x). If $(x_1, \ldots, x_n)$ is a system of local coordinates at x, the matrix of $D_f(x)$ is the 1 x n-matrix:

$$\left(\frac{\partial f}{\partial x_1}, \ldots, \frac{\partial f}{\partial x_n}\right)$$

We say that x is *a critical point* of f if $D_f(x)$ is not of maximal rank 1 at x, i. e. if all partial derivatives $\partial f/\partial x_i$ vanish at x. $y = f(x)$ is then called a *critical value*. If y is not a critical value (i. e. is a regular value), then f is 'locally trivial'. That means:

i) the inverse image $f^{-1}(y)$ of y is a submanifold M_y of M of codimension 1 (i. e. of dimension n-1), and
ii) for a sufficient small neighbourhood U of y, $M_U := f^{-1}(U)$ is diffeomorph to the cartesian product $M_y \times U$ (see fig. 11).

If x is a critical point, its degree of criticalness is measured by the degree of degeneracy of the higher derivatives of f at x. The second deriva-

3 For mathematical precisions see for instance Thom 1972, 1980, Zeeman 1977, Golubitsky & Guillemin 1973, Chenciner 1973, 1980, Petitot 1982.

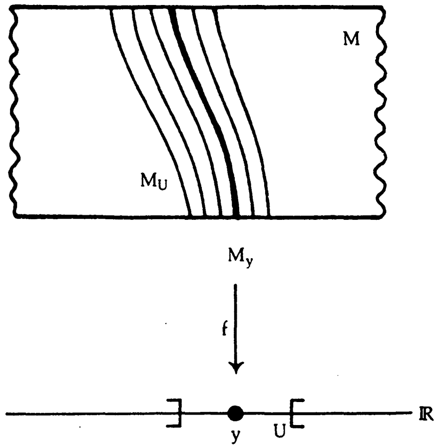


Figure 11. The potential $f: M \to \mathbb{R}$ is locally trivial.

tive of f is given by the symmetric (n,n)-matrix of the second partial derivatives:

$$H = \left(\frac{\partial f}{\partial_{xi}\, \partial_{xj}} \right) \quad 1 \leq i, j \leq n$$

If H is of maximal rank n, one says that x is a *non-degenerated* critical point, and one defines its *index* as the index of the quadratic form H. The critical elements of f are its critical points with their degree of criticalness and its critical values with their multiplicity. These are invariants of the differentiable type.

3.2 The Criterion of Structural Stability

A potential $f \in \mathscr{F}$ is structurally stable if every potential g sufficiently close to f (for the differentiable topology) is G-equivalent to f, in other words, if the orbit $\tilde{f}$ of f contains a full neighbourhood of f. A fundamental theorem, the Morse's theorem, asserts that, if M is compact, f is structurally stable if and only if:

i) all of its critical points are not degenerated, and
ii) all of its critical values are distinct.

Therefore there are only two possible reasons for instability: degeneracy of critical points (bifurcation catastrophes), and equality of critical values (conflict catastrophes).

3.3 Genericity and Transversality

The equivalent of Taylor's series for a potential f: $M \to \mathbb{R}$ is given by mappings $j^k f: M \to J^k$, which associate to each $x \in M$ the sequence $j^k f(x)$ of the derivatives of order $\leqq k$ of f at x (these sequences are called 'k-jets' and generate fibered spaces J^k with base space $M \times \mathbb{R}$). Now it is quite easy to see that the first condition of Morse's theorem can be expressed by saying that the 1-jet $j^1 f(x)$ of f at x is *transversal* on the zero section of J^1 (j^1 f(x) = 0 implies that x is critical and transversality implies the non-degeneracy). The second condition can likewise be expressed in terms of transversality. According to a theorem of R. Thom this implies that structural stability is a *generic* property of potentials, i. e. that instability is 'exceptional'.

3.4 Finite Determination, Universal Unfoldings and Transverse Models

Let $K_{\mathscr{F}}$ be the global catastrophic set, inherent in $\mathscr{F}$, of structurally unstable potentials. If $f \in K_{\mathscr{F}}$, the geometry of $K_{\mathscr{F}}$ in a neighbourhood of f classifies the stable types that can be obtained from f by small deformations. If f is 'infinitely' unstable (for instance if f is constant in the neighbourhood of a point $x \in M$), the local geometry of $K_{\mathscr{F}}$ at f will be 'chaotic'. The elementary theory is interested in the cases of 'weak' instability, where the following conditions are satisfied:

i) f is of *finite determination*, i. e. equivalent to one of its jets of finite order (which is a polynomial). This implies that f is of *finite codimension*

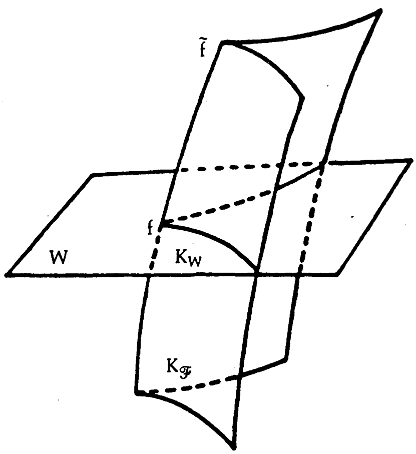


Figure 12. The local structure of $\mathscr{F}$ near a potential f which is weakly unstable.

which means that the orbit $\tilde{f}$ of f admits at f a finite dimensional complementary space W in $\mathscr{F}$.

ii) In a neighbourhood of f, the pair $(\mathscr{F}, K_{\mathscr{F}})$ is equivalent to the direct product of the pair (W, K) (where $K = K_{\mathscr{F}} \cap W$), with the orbit $\tilde{f}$ (see fig. 12).

In this case we say that (W, K) is a *transverse model* of f. W being of finite dimension k, it is isomorphic with a neighbourhood W' of the origin of $\mathbb{R}^k$. One can then interprete W as a field (an unfolding) $\sigma : W' \rightarrow \mathscr{F}$ which associates to $w \in W'$ the corresponding element $f_w \in W$. σ is called a *universal unfolding* of f, and f is called its *organizing centre*. The theorem of the universal unfolding essentially says that σ is *unique* up to equivalence and allows to reconstruct *all* the unfoldings of f.

3.5 *Degrees of Instability and Stratifications*

If $f \in K_{\mathscr{F}}$ satisfies to the preceding elementary situation, the possibility of progressively stabilizing it by successive steps is readable in the *geometry* of K (where (W, K) is a universal unfolding): K will be a *stratification*, which means a 'pile' of spaces with singular loci of decreasing dimensions, each stratum corresponding to a precise degree of instability. These stratifications geometrize the concept of classification.

3.6 *Normal Forms and the Theorem of Classification*

Thom's theorem on elementary catastrophes classifies (up to differentiable equivalence) the singularities of a potential f which are of codimension $\leqslant 4$. For every case it yields:

i) the number of internal variables (1 or 2) which intervene effectively in the instability of f;
ii) a normal form (polynomial) of the singularity, i. e. the simplest representative of its equivalence-class;
iii) a normal form of its universal unfolding.

3.7 *Methodological Rules for the Modelling*

Given a phenomenon, which manifests itself as a categorization K of a control space W, the methodology of catastrophe theory consists, as we have seen, in abducing constraints on the generative mechanism X from this phenomenology. It is determined by certain principles, of which we give two examples here.

(a) The phenomenology of elementary catastrophes being known, it is possible, if one comes across one of these empirically, to make the hypothesis that the associated catastrophe C governs the phenomenon and is an 'infrastructure' of it, in other words that the real generating mechanism X is a complexification of C.

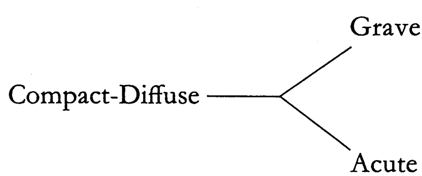
(b) On the other hand, given a field $\sigma : W \rightarrow \mathcal{H}$ one can assume *a priori* that it is structurally stable, in so far as it does actually exist. This stability is expressed by a condition of *transversality* of σ with respect to $K_{\mathcal{H}}$. Now the consequence of this transversality is that σ (W) *has to avoid* the strata of $K_{\mathcal{H}}$ whose codimension is higher than the dimension of W. This is a fundamental principle: *the dimension of the control space W drastically bounds the complexity of the morphologies which can unfold in a structurally stable way.*

4. Categorical Perception and Elementary Catastrophes

Since categorical perception, which transforms the phonetic flux into phonological sequences, is a perceptual case of a critical phenomenon, it is, as we have already seen, quite natural to apply to it the catastrophic methodology. At the moment, these methods cannot be developed very far, for lack of precise experimental results. Let us, nevertheless, propose some programmatic indications.

4.1 The Classification of Vowels

Although being perceived rather continuously in isolated state, vowels are perceived in a far more categorical way if they are embedded in a phonetic flux. Let us reconsider their classical phonological description in terms of distinctive features



Acoustically speaking, this description is traditionally done in the following manner:

i) In so far as they are stationary states, the vowels are characterized by the frequency and the intensity of their formants (quantitative parameters);
ii) The first two formants F_1 and F_2 are essential and one can confine one's attention to them as a first approximation;
iii) Qualitatively speaking (i. e. on the level of the phonological perception of higher order spectral configurations), the compactness of /a/ is due to the regrouping of F_1 and F_2 in the central zone of the spectrum;
iv) On the contrary, the diffuse character of /i/ and /u/ is due to the separation of F_1 and F_2; the acute character of /i/ is due to the dominance of F_2 over F_1 and the grave character of /u/ to the dominance of F_1 over F_2.

One can see that such a qualitative description derives from the following idealized situation:

i) A control space W of dimension 2 having as coordinates the frequencies ν_1 and ν_2 of F_1 and F_2, controls the continuous spectra with one or two peaks;
ii) the compactness corresponds to the domain of one peak, the diffuse character to the domain of two peaks and the opposition "acute/grave" to the two possibilities of dominance of one peak over the other (see Fig. 13a).

We do find here again the categorizing morphology of the 'critical point' type (in the sense of phase transitions) that we have already seen in 2.6.2. Let us recall that it *does not* fulfill the discreteness condition.

It is thus quite natural to advance the hypothesis that the auditive transformation T:S → F (cf. 2.6.1) transforms the acoustic situation into the idealized situation of Figure 13b.[4]

Now the auditivly idealized situation is typically catastrophic: if one substitutes the spectral peaks with the minima of a potential, it corresponds to the best known elementary catastrophe, viz. the cusp catastrophe. The cusp catastrophe unfolds a singularity of codimension 2 of type x^4, which

4 This hypothesis fits with experimental data. See for instance Chistovitch 1985 and Schwartz 1987.

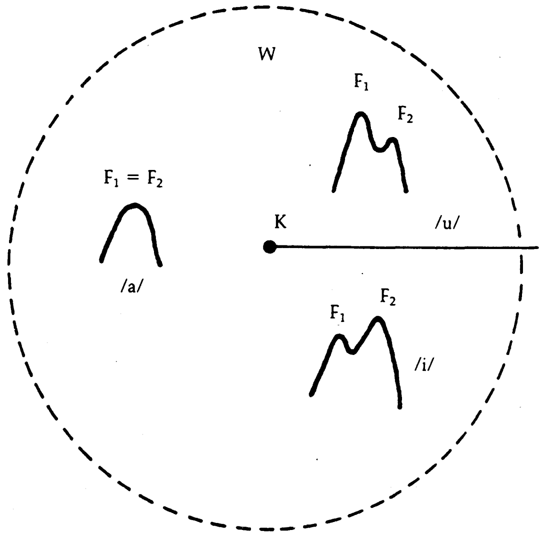


Figure 13a. The fundamental vocalic triangle as a categorization of the type "critical point".

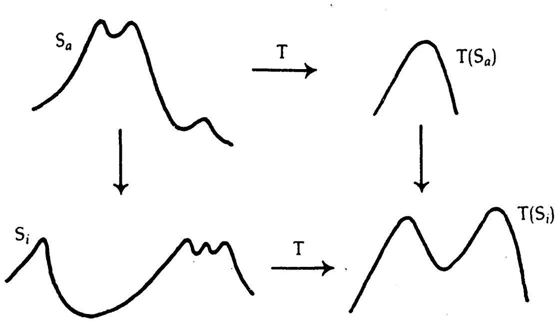


Figure 13b. The transformation T expresses the gross qualitative structure of the /a/ and /i/ spectra S_a and S_i.

means a degenerated minimum (of a potential defined on an internal space $M = \mathbb{R}$ with internal coordinate x) obtained by the collapse of two minima and one maximum all non-degenerated. The classification theorem, already mentioned in 3.6, shows that its universal unfolding is given by

(1) $f_w = x^4 + ux^2 + vx,$

the coordinates u and v ranging through a neighbourhood W of the origin of $\mathbb{R}^2$ (external control space).

It is easy to verify that the catastrophic set K of W defined as the set of all $w \in W$ for which f_w has (according to Morse's theorem) degenerated critical points or multiple critical values is that which is represented in figure 14(a). If one plots on a third axis (with coordinate x) the critical points of f_w, one obtains the famous folded surface shown in Figure 14(b).

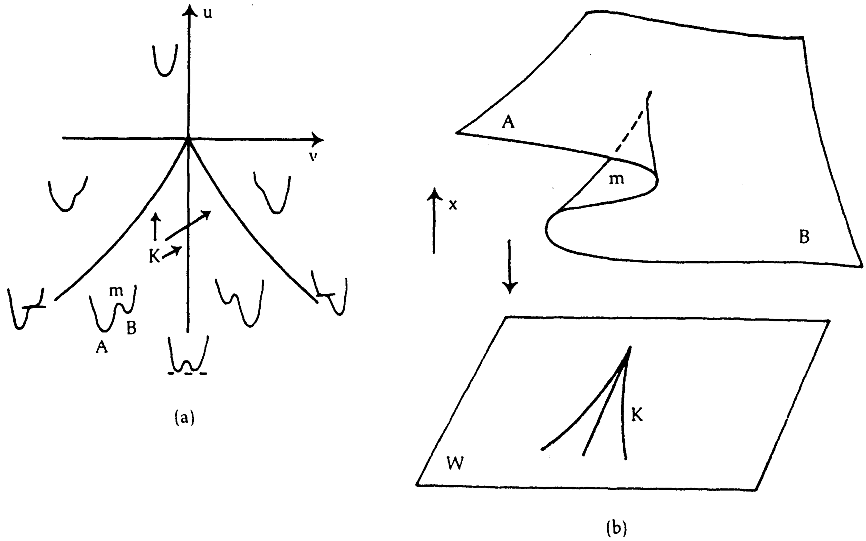
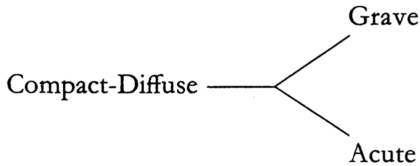


Figure 14a, b. The cusp catastrophe.

One can therefore set up the hypothesis that, after the auditive transformation, the interface /i/ / /u/ is a *symmetric* interface induced by a conflict catastrophe, whereas the interfaces /a/ / /u/ and /a/ / /i/ are *asymmetric* interfaces induced by bifurcation catastrophes. This categorization of 'critical point' type is radically opposed to those of the 'triple point' type decomposing W into three domains by means of three conflict strata. In so far as these two categorizations are the most typical which can appear in a stable way in an external space of dimension 2, it will be essential to test them experimentally. Yet, be that as it may, the catastrophic methodology allows to state *a priori* that if the description:

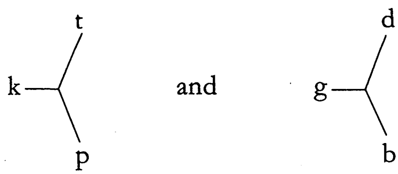


is valid, a cusp catastrophe is necessarily involved in the triangle of the cardinal vowels.

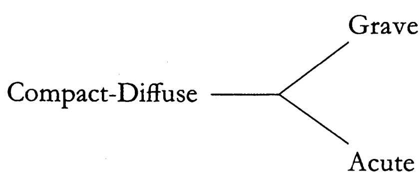
From there on, one should be able to take into account the complexifications of that universal categorization based on K_0 and to retrieve the classification of the vocalic systems (see Crothers, 1978). In order to do that one has to take into consideration the third formant F_3 and therefore to introduce the elementary catastrophe, called "butterfly", classifying the potentials with three minima and two maxima. This catastrophe with organizing centre x^6 has codimension 4. Its geometry is already of a notable complexity and suffices, in our opinion, to geometrize the vocalic hierarchies of various languages (see Petitot, 1982 and 1985).

4.2 *Classification, Hierarchy and Stratification of the Stop Consonants*

One knows that the triangles of stop consonants:



are described by the hierarchy:



One can justify it by hypothesizing, as Stevens did, that a stop consonant is essentially characterized by its spectral onset conceived as a higher order spectral configuration (Stevens and Blumstein, 1978). Yet, if this is the case, it will again be a *cusp* catastrophe which organizes the two consonant triangles. Now, Stevens and Blumstein have experimentally found categorizations of the 'triple point' type. Here there is a discrepancy that needs to be resolved.

Be that as it may, let us show what the methodology of catastrophe theory can contribute to a precise model of the classification of stop consonants. As a test we have chosen a model proposed by D. Massaro and G. Oden.

4.2.1 The Massaro-Oden Model

In the two articles which we will use as guiding line (Oden and Massaro, 1978, Massaro and Oden 1980) the authors proposed and tested a model of integration of some acoustic cues in a unified percept. The two cues studied were those of voicing and point of articulation for the stops. They range in the control space (v, a) with coordinates 'v' for the VOT and 'a' for the point of articulation. The hypotheses are the following:

- i) The acoustic features are detected and perceived independently of each other.
- ii) Each feature is evaluated by measuring its degree of presence in the signal.
- iii) Each phoneme is defined by a prototype propositionally stored in the long-term memory (integration of the acoustic information).
- iv) Each sound is identified on the basis of the evaluation of its features in comparison to the competing prototypes.

Let s be a phonetic stimulus, in this case a stop consonant, and let v and a be the two considered features. According to Massaro and Oden, the features are attributes of s, which are not binary but gradient and which consequently are described by fuzzy predicates. In other words, if one considers the oppositions voiced/$\sim$voiced and labial/$\sim$labial (alveolar) one associates with them the fuzzy predicates V and L and one will have to use the rules of the fuzzy predicate calculus:

- i) $V(s), L(s) \in [0, 1]$
- ii) $\sim V(s) = 1 - V(s)$, etc.
- iii) $V \wedge L(s) = V(s) \cdot L(s)$, etc.

In a first elaboration of their model, Massaro and Oden simply identified the prototypes of stops with the fuzzy predicates, which translates their Boolean characterization in terms of distinctive features ($\pm$, $\pm$). They have therefore used the correspondences:

$$/b/ \Leftrightarrow B = V \cdot L$$
$$/d/ \Leftrightarrow D = V \cdot (1-L)$$
$$/p/ \Leftrightarrow P = (1-V) \cdot L$$
$$/t/ \Leftrightarrow T = (1-V) \cdot (1-L)$$

If $/\alpha/$ is one of the prototypes considered and if one defines the probability that the stimulus s is identified with $/\alpha/$ as the conditional probability

$$p(|\alpha|/s) = \frac{A(s)}{B(s) + D(s) + P(s) + T(s)}$$

where A(s) is the predicate associated with $/\alpha/$, one simply obtains $p(\backslash\alpha\backslash/s) = A(s)$ because $VL + V(1-L) + (1-V)L + (1-V)(1-L) = 1$.

It is easy to calculate what this elementary model predicts. Let us place ourselves in the control plane $W = (v, a)$, $v, a \in [0, 1]$. We can identify the evaluation of s with its position in W, because $V(s) = v(s)$ and $L(s) = a(s)$. Given a point s of W, we calculate the values at s of the four 'predicates' (i. e. of the four polynomes of the second degree) $v.a.$, $v(1-a)$, $(1-v)a$ and $(1-v)(1-a)$ and we suppose that s belongs to the category that gives the greatest value. The boundary K, categorizing W, is therefore constituted by the (singular) points of W where there is a competition between two predicates and it is defined by the equations:

$$va = v(1-a) \text{ i.e. } a = 1/2 \text{ or } v = o$$
$$va = (1-v)a \text{ i.e. } v = 1/2 \text{ or } a = o$$
$$va = (1-v)(1-a) \text{ i.e. } v + a = 1$$
$$v(1-a) = (1-v)a \text{ i.e. } v = a$$
$$v(1-a) = (1-v)(1-a) \text{ i.e. } v = 1/2 \text{ or } a = 1$$
$$(1-v)a = (1-v)(1-a) \text{ i.e. } a = 1/2 \text{ or } v = 1$$

The four lines $v = 1/2$, $a = 1/2$, $v + a = 1$ and $v = a$ divide W into eight domains and it is trivial to calculate the order of B, D, P and T in each of these domains from the inequalities:

$$v \gtrless \tfrac{1}{2} \Leftrightarrow B \gtrless P \text{ and } D \gtrless T$$
$$a \gtrless \tfrac{1}{2} \Leftrightarrow B \gtrless D \text{ and } P \gtrless T$$
$$v + a \gtrless 1 \Leftrightarrow B \gtrless T$$
$$a \gtrless a \Leftrightarrow D \gtrless P \qquad \text{(see fig. 15)}$$

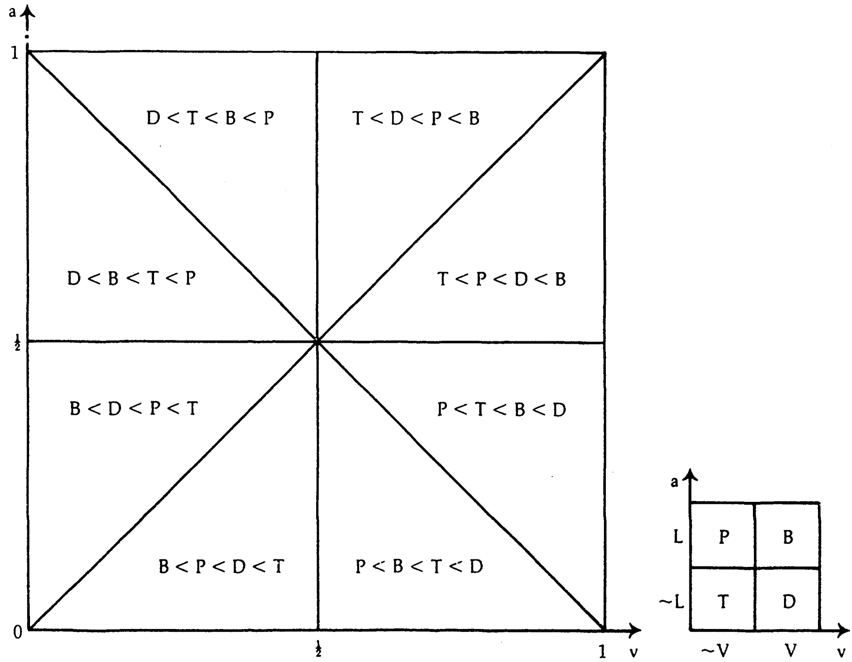


Figure 15. The categorization induced by the Massaro–Oden's elementary model.

On the other hand, the model predicts that the probability curves p(|α|/s) as functions of 'a' with 'v' fixed, for example, are straight lines. Yet, the tests of identification do not verify this prediction (see fig. 16).

This discrepancy has lead Massaro and Oden to complexify their model by supposing that the fuzzy predicates defining the prototypes are not simply the translations of their description in terms of distinctive features, but more general *functions* of these features, for instance of the form $A = V^p L^q$, or $A = (1 - V)^p L^q$, or $A = V^p + L^q$ etc. The problem is now to find the functions which correspond best with the experimental results (see fig. 17).

Such complexifications of the model lead to a transformation of the boundary K (see Fig. 18), and it is on this last point that we want to make some remarks. This model raises indeed problems in catastrophe theory, which will be interesting to develop. Its general idea is the following. One considers a control space W whose coordinates $w = (u_1, \ldots, u_r)$ (here (v, a)) are values of acoustic cues. Then one considers n prototypes, defined by n

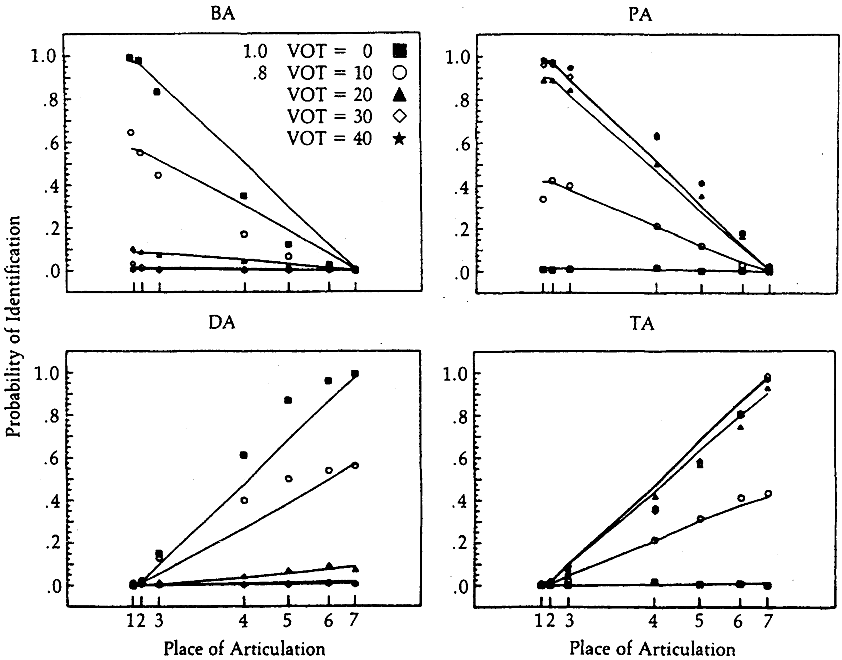


Figure 16. From Oden and Massaro (1978: 181). Identification probabilities and predictions of the simple fuzzy logical model. (Each panel presents the data for a given response alternative. Note that the spacing along the abscissa is proportional to the spacing of the subjective place values. VOT = voice onset time).

functions P_1 (w),..., P_n (w) (here B, D, P and T). These prototypes define a field σ:

$$w \to \mathbb{R}^n$$
$$w \to (P_1, \ldots P_n).$$

In $\mathbb{R}^n$ the equations $P_i = P_j$ $(i \neq j)$ determine a stratification K_0, which is the inverse image by the mapping

$$\gamma_n\colon\ \mathbb{R}^n \to \mathbb{R}^N$$
$$(P_i) \to (P_i - P_j)\ i > j \text{ of the stratification } K^* \text{ of } \mathbb{R}^N\ N = \frac{n(n-1)}{2}$$

defined by the coordinate hyperplanes. K is then nothing else but the inverse image of K_0 by σ i.e. of K^* by $\gamma_n\sigma$. The 'simple' model derives from a field $\sigma_0 : W \to \mathbb{R}^n$ *which is not transversal* on the stratification K_0. This structurally

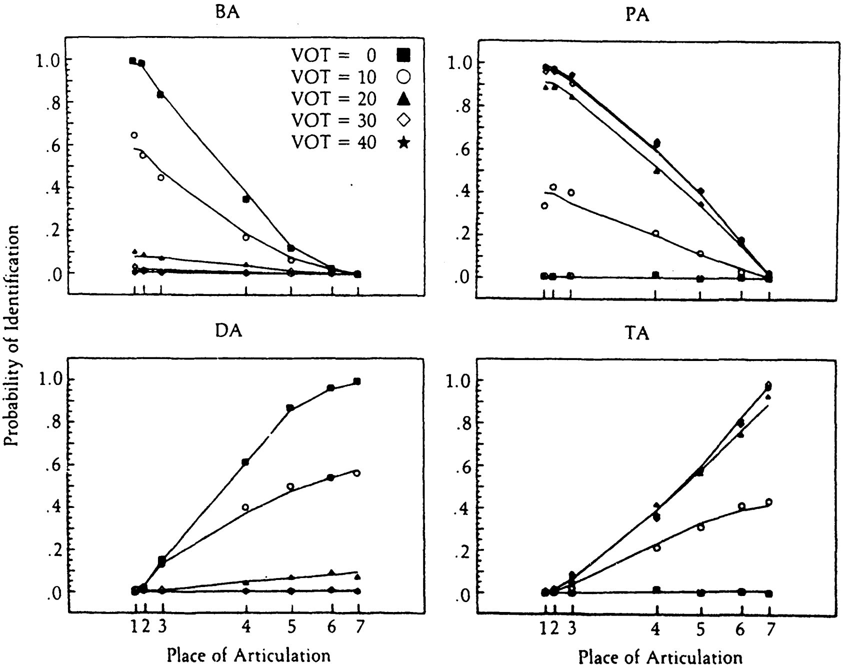
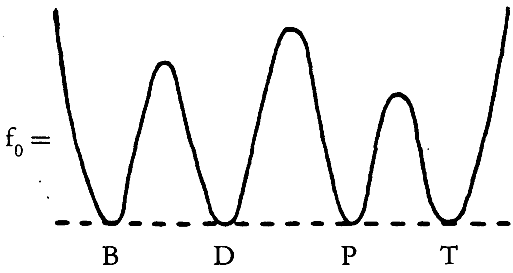


Figure 17. From Oden and Massaro (1978: 183). Identification probabilities and predictions of the complex fuzzy logical model. (Note the spacing along the abscissa is proportional to the spacing of the subjective place values. VOT = voice onset time).

unstable situation is taken as 'organizing centre' of 'complex' *stable* models where σ becomes transversal on K_0 and therefore *avoids* the strata of K_0 of codimension $> r$ (see 3.7). This remark is epistemologically and methodologically important because it shows that it is useless to look for an *exact* expression of the functions P_i predicatively characterizing the prototypes. The good methodology is to start from the *non-generic* situation σ_0, associated with the Boolean description (in the "fuzzy" sense) of the prototypes, and to *stabilize* it by perturbing it. Once again it is the boundaries K which are important and not the exact form of the functions which induce them.

As to that stabilization of σ_0 one can make another additional remark. The generic boundary K obtained in the 'complex' model (see fig. 18) is a typical example of a plane section in the unfolding of a *quadruple* point. This suggests to use a quadruple point singularity as organizing centre of the situation, for instance the singularity:



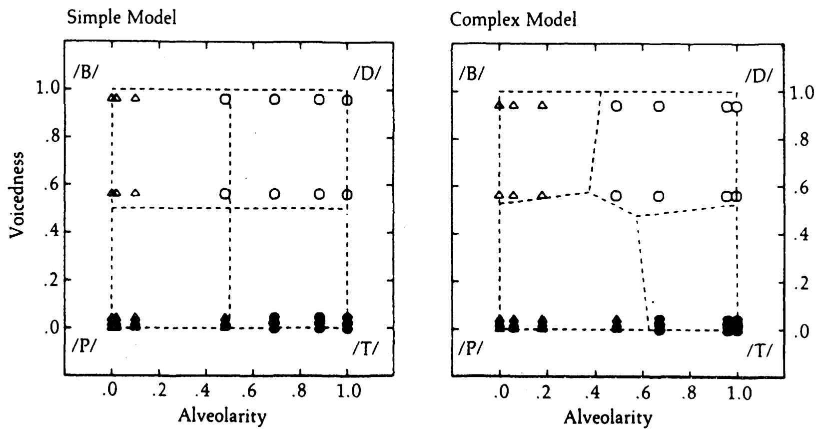


Figure 18. The transformation of the K boundary when one complexifies the fuzzy predicates defining the prototypes of stop consonants. From Oden and Massaro (1978: 185). The dashed lines partition the parameter spaces into regions in which a given response alternative is predicted by the respective model to be most likely. Each point represents the subjective position of a stimulus and specifies what phoneme it was most often identified to be, with open triangles, closed triangles, open circles, and closed circles standing for /b/, /p/, /d/, and /t/, respectively.

Such a singularity is of codimension 3 and it is clear that its partially or completely stabilized deformations are exactly classified by the situations $X_1 \leq X_2 \leq X_3 \leq X_4$ where (X_1, X_2, X_3, X_4) is a permutation of (B, D, P, T). In other words, if (Λ, Γ) is the universal unfolding of f_0 and if one considers the field $\psi : \Lambda \rightarrow \mathbb{R}^4$ associating to f_λ the values of its minima, the catastrophic set Γ is the inverse image of K^* by the mapping $\gamma_{40}\psi$.

It would therefore be interesting to know if the field $\sigma : W \rightarrow \mathbb{R}^4$ sought by Massaro and Oden can be *factorized* through ψ. Such a factorization is far from being evident as far as it presupposes that one can associate a potential f_s with four minima to each stimulus s of W, potential whose values would be the evaluations B(s), D(s), P(s) and T(s). Now such a global function (which can always be introduced abstractly) would be defined on an internal space which is not easily interpretable.[5]

Let us, nevertheless, accept the idea of a factorization. One can then visualize the stabilization of σ_0 much better. Indeed σ_0, described in figure 15, corresponds to a *non-generic* plane section (non-generic because it goes through the organizing centre, which is of codimension 3 > 2) of the universal unfolding (Λ, Γ) of f_0. This section is shown in figure 19.

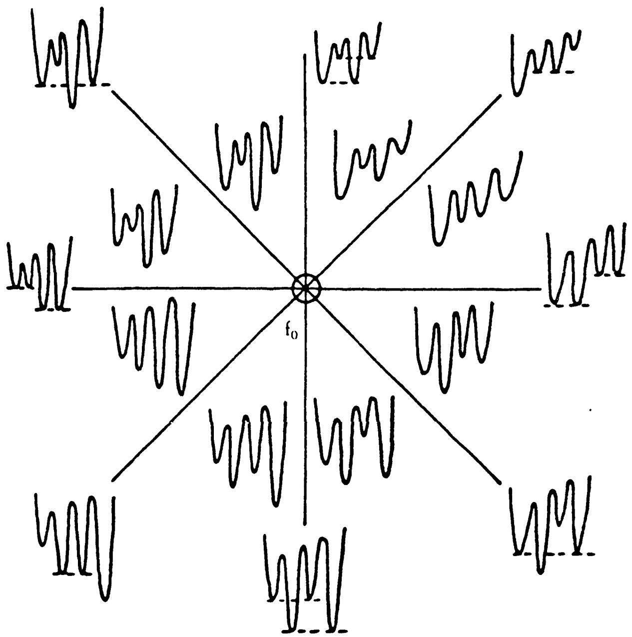


Figure 19. The elementary model of figure 15 is a non generic section of the universal unfolding of a quadruple point f_0.

The non-genericity of σ_0 becomes evident once it is so expressed. W being of dimension 2, it can unfold in a stable way only the conflict catastrophes of complexity lower to that of triple points. This is not the case for σ_0. In

5 In the connectionist point of view, the potential is a *harmony function* in the sense of Smolensky.

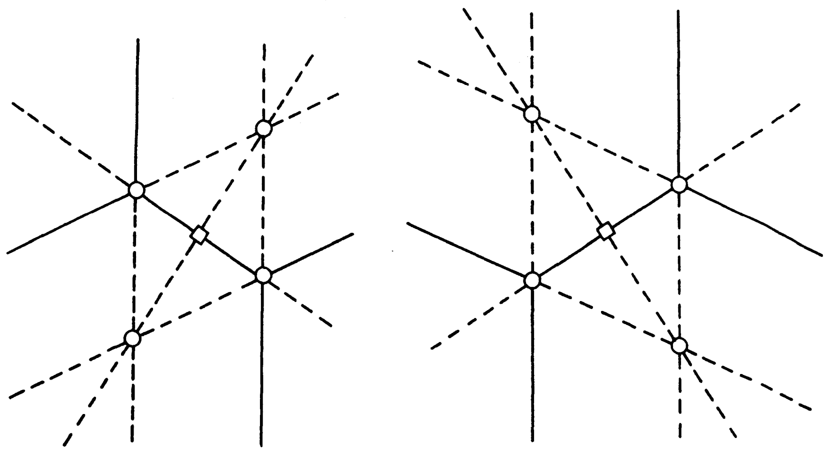


Figure 20. The two stabilizations of the non generic section of figure 19. Full lines represent Maxwell strata, circle represent triple points and squares represent double conflict points.

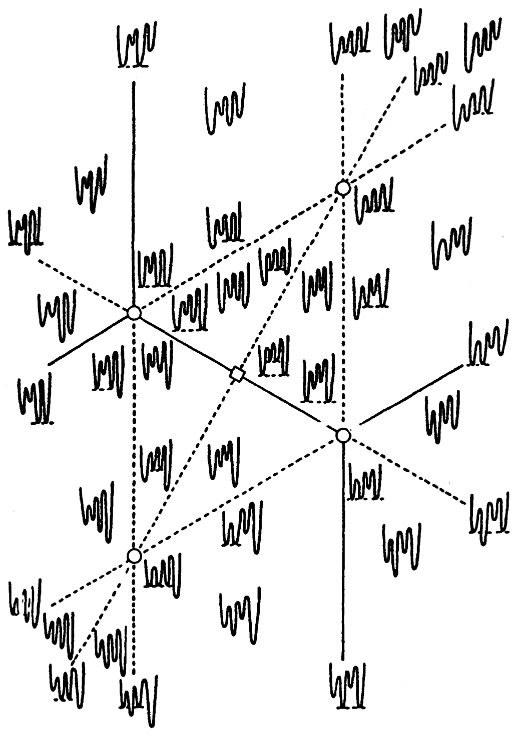


Figure 21 (a)

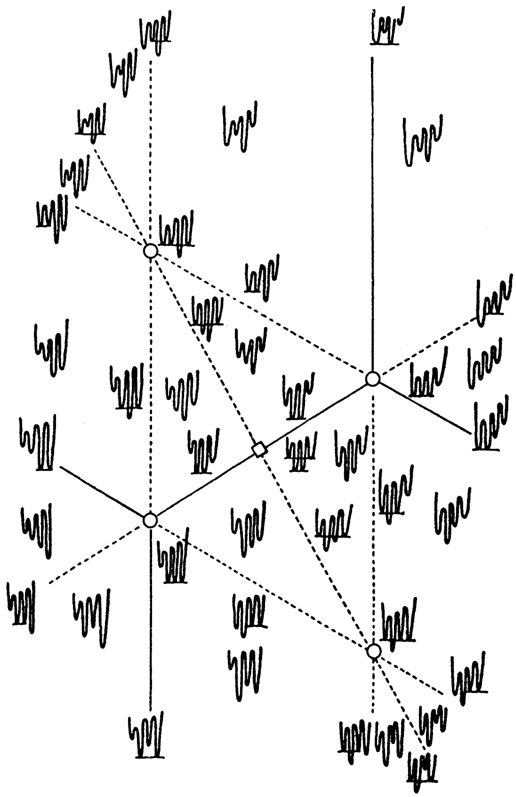


Figure 21 (b)

(W, K) the coordinate axes 'v' and 'a' are *lines* of double conflicts. But, because the double conflicts are of codimension 2, they can appear in a stable way in dimension 2 only as *isolated* points.

As σ_0 stabilizes itself, W becomes a generic section of (Λ, Γ), the quadruple point breaks into four triple points and one point of double conflict and the axes split according to the two possibilities shown in figure 20. By applying what is called the Maxwell convention, one gets the catastrophic set of the 'complex' model of Massaro – Oden. In figure 21, one finds the classification of the differentiable types induced by the two types of generic sections of figure 20.

4.2.2 Stratification of Boundaries and Consonant Hierarchy

Independently from the computational content of the cited model, the experiments of Massaro and Oden show that in the plane (v, a), the system of boundaries transforming the stops /b/, /d/, /p/ and /t/ into *positional* values, is a generic section of a quadruple point, which means an archetype of decomposition of a plane into four domains by conflict strata. In order to be complete, one will have to extend the experiments to the set of the *six* basic stops /b-d-g/ and /p-t-k/. One might think that in doing so one will obtain a catastrophic set gluing together two models, namely, that of /b, d, p, t/ on the one hand, and that of /d, t, g, k/ on the other (see figure 22), (which will show moreover on what account the experiments of Lisker and Abramson, as we have already mentioned, have been insufficient).

The *geometry* of the system of boundaries can provide precious information about the hierarchical relations that stop consonants maintain with each other. The fact that in the model of Massaro and Oden, the domains of /p/ and /d/ are *adjacent*, whereas those of /b/ and /t/ are *separated*, indicates that the contrast between /b/ and /t/ is much greater than that between /p/ and /d/.

Though naive, this remark might perhaps allow to interprete the hierarchical *phonological* universals appearing in the system of the stops (or fricatives, or vowels), in terms of *phonetic* stratification (and thus in terms of the phenomenology of substance). In this way a catastrophe theory of *marking* phenomena could be developed.

Thomas Gamkrelidze has shown for instance that if voicing is relevant for the stops in a phonological system then (1) the /b/ (voiced + labial) dominates the /g/ (voiced + velar) and (2) the /k/ (unvoiced + velar) dominates the /p/ (unvoiced + labial) (Gamkrelidze 1972 and 1978). These relations of dominance manifest a phonological universal according to which, if a system of stops is incomplete, it is always the *recessive* stops /p/ and /g/ which are lacking.

One might think that such a universal is nothing but the *phonological* manifestation of the *geometry* of the catastrophic set in figure 22. If such a hypothesis could be confirmed, categorical perception would not confine itself to *locally* unify the audio-acoustic and phonological levels with respect to the notion of distinctive feature. It would also unify them *globally* with respect to the notion of phonological system. Via the catastrophic paradigm a phonological system would then appear like a global system of phonetic boundaries, whose geometrical stratification expresses the hierarchical relations of marking and dominance.

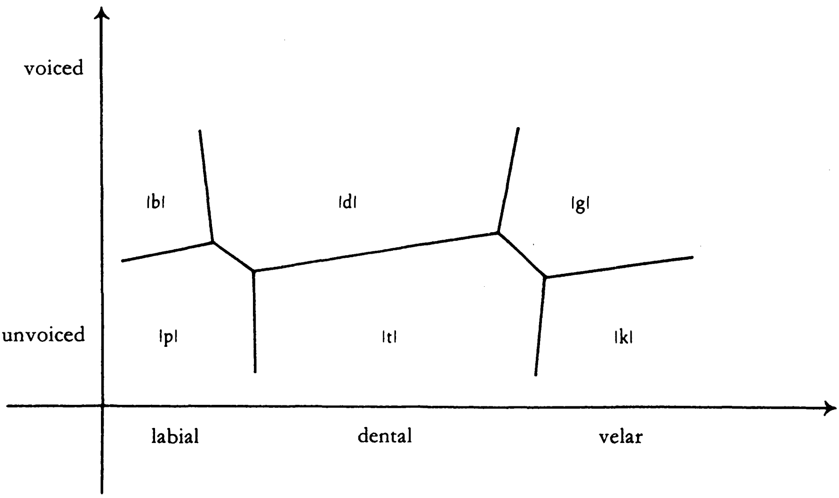


Figure 22. A rough proposition for the boundaries of stop consonants.

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